Math 6, SchoolNova Homework 5

(1) Syllogisms A syllogism is a collection of logical premises and a conclusion that may be deduced from them. It is a very simple kind of proof. A famous example is:

- All men are mortal (first premise)
- Socrates is a man. (second premise)
- Therefore, Socrates is mortal (conclusion)

Not all syllogisms have exactly this form, but they all employ similar reasoning. Here are some examples from Lewis Carroll, author of Alice in Wonderland, who was also an enthusiast and creator of logic puzzles:

- (a) No birds, except ostriches, are 9 feet high.
- (b) There are no birds in this aviary that belong to anyone but me.
- (c) No ostrich lives on mince pies.
- (d) I have no birds less than 9 feet high.
- (2) Another one of Lewis Carroll's puzzles:
  - (a) My saucepans are the only things I have that are made of tin.
  - (b) I find all your presents very useful.
  - (c) None of my saucepans are of the slightest use.
- (3) Another one of Lewis Carroll's puzzles:
  - (a) No interesting poems are unpopular among people of real taste.
  - (b) No modern poetry is free from affectation.
  - (c) All your poems are on the subject of soap-bubbles.
  - (d) No affected poetry is popular among people of real taste.
  - (e) No ancient poem is on the subject of soap-bubbles.
- (4) Write the truth table for the formula: (NOT P) AND (Q OR (NOT R))

(5) If Alex comes to a party, Betsy will not come. Alex never comes to a party where Charley comes. And either Betsy or Charley (or both) will certainly come to the party.

Based on all of this, can you explain why it is impossible that Alex comes to the party?

- (6) Let
  - A=set of all people who know French
  - B=set of all people who know German
  - C=set of all people who know Russian

Describe in words the following sets:

- (a)  $A \cap B$
- $(b)A \cup (B \cap C)$
- (c)  $(A \cap B) \cup (A \cap C)$
- (d)  $C \cap \overline{A}$ .

- (7) Let  $A = \{1, 3, 5\}, B = \{3, 5, 6\}$ , and  $C = \{1, 3, 7\}$ . Calculate the sets in the right hand side and in the left hand sides of the following expressions, and show that the results match:
  - (a)  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
  - (b)  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(8) Let us take the usual deck of cards. As you know, there are 4 suits, hearts, diamonds, spades and clubs, 13 cards in each suit.

Denote:

- *H*=set of all hearts cards
- Q=set of all queens
- R=set of all red cards

Describe by formulas (such as  $H \cap Q$ ) the following sets:

- all red queens
- all black cards
- all cards that are either hearts or a queen
- all cards other than red queens

How many cards are there in each set?

(9) In a class of 25 students, 10 students know French, 5 students know Russian, and 12 know neither. How many students know both Russian and French?