(1) In this problem, you have to express your answer as a simplified exponent (you do not have to compute numerically the expressions that you find). Simplify the expressions below using the power laws: $(a \times b)^n = a^n \times b^n$, $(a^n)^m = a^{n \times m}$, $a^n a^m = a^{n+m}$, $a^n / a^m = a^{n-m}$, $a^{-n} = 1/a^n$ and $a^0 = 1$.

(a) $\frac{2^5 4^4}{2^7}$ (b) $6^5 \times 3^{-4}$ (c) $\frac{5^{-2}}{5^{-4}}$

(2) You have 4 shirts and 4 ties colored red, yellow, blue and green. How many shirt and tie combinations are there if you refuse to wear a shirt and a tie of the same color?

(3) A sly elementary school teacher decides to play favorites without telling anyone. If they have 15 students in their class, in how many ways can they choose a favorite student, a second favorite student, and a third favorite student?

(4) Compute $\frac{5!}{4!}$, $5! - 4!$, ${}_3P_2$, ${}_3P_3$

(5) (a) How many ways are there to draw 3 cards from a 52-card deck? (Order matters: drawing first king of spades, then queen of hearts is different from drawing them in opposite order).

(b) How many ways are there to draw 3 cards from a 52-card deck if after each drawing we record the card we got, then return the card to the deck and reshuffle the deck? (As before, order matters.)

(6) A puzzle consists of 9 small square pieces which must be put together to form a 3×3 square so that the pattern matches (this kind of puzzles is actually quite hard to solve!). It is known that there is only one correct solution. If you started trying all possible combinations at random, doing one new combination a second, how long will it take you to try them all?

(7) Polly is talking parrot who speaks in 3-word sentences. A Polly's sentence always starts with a pronoun, which is followed by a verb, and then by a noun. Polly knows:
- 2 pronouns: I and WE
- 3 verbs: LOVE, WANT, and COOK,
- 4 nouns: FOOD, CRACKER, FRIEND, and SHMOLLAR

Polly's friend Dolly the parrot can talk as well. A Dolly's sentence always starts with an adjective, which is followed by a noun, and then by a verb:
- 3 adjectives: HAPPY, HUNGRY, and LONELY,
- 2 nouns: PARROT and CROCODILE,
- 3 verbs: SINGS, CRIES, and WORKS.

(a) How many different phrases can Polly the parrot say?
(b) How many different sentences can Dolly the parrot say?
(c) Polly and Dolly are creating a two-phrase story. Each parrot contributes a sentence. How many different stories can they come up with?

(8) Going back to the example with the animal habitats, how many different possible arrangements can you have if you have 6 animals and 15 habitats?

(9) A group of 6 club members always dine at the same table in the club; there are exactly 6 chairs at the table. They decided that each day, they want to seat in a different order. Can they keep this for a year? Two years?

(10) How many ways are there to seat 15 students in a classroom which has 15 chairs? If the room has 25 chairs?

(11) 10 people must form a circle for some dance. In how many ways can they do this?