

- (1) Prove Theorems 21–34 in Book IX of Euclid’s Elements (they are Pythagorean in origin):

(IX .21) A sum of even numbers is even.

(IX .27) Odd less odd is even.

- (2) For tax purposes, Ancient Greeks often approximated the area of a quadrilateral field by multiplying the averages of the two pairs of opposite sides. For example, opposite sides may be given by

$$s_1 = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} \quad \text{opposite} \quad s_2 = \frac{1}{8} + \frac{1}{16}$$

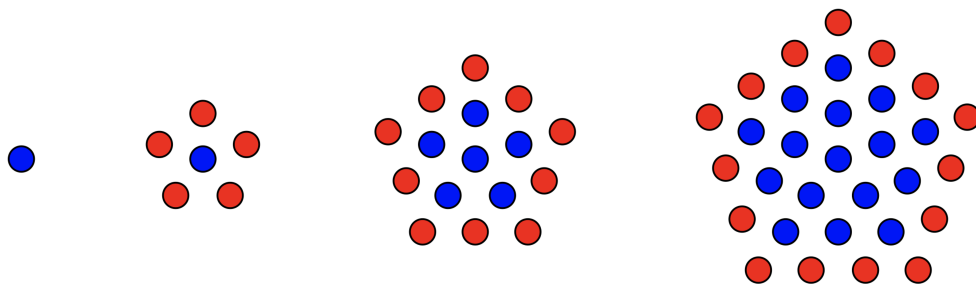
or

$$\sigma_1 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \quad \text{opposite} \quad \sigma_2 = 1$$

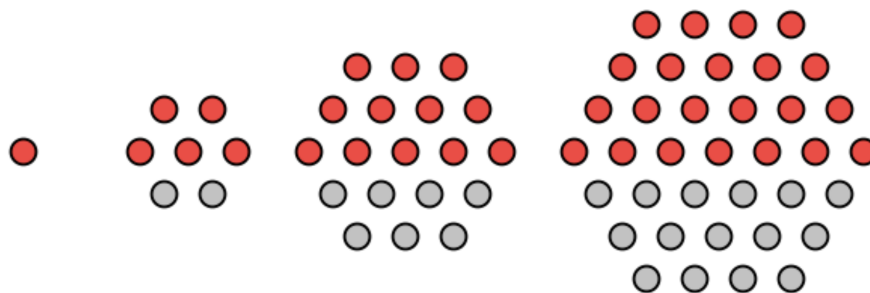
where all lengths $s_1, s_2, \sigma_1, \sigma_2$ are in fractions of a *schonion*, approximately 150 feet. Find the approximation for both plots of land, in terms of square *schonion* (round up, as any good taxman would).

- (3) Show that triangular numbers satisfy $T_n^2 + T_{n+1}^2 = T_{(n+1)^2}$.

- (4) Develop a theory for the centered pentagonal numbers:



- (5) Develop a theory for the centered hexagonal numbers. Do they look familiar?



- (6) A boy had a bag of apples. He gave $\frac{1}{2}$ of them to his parents, $\frac{1}{5}$ to his brother, $\frac{1}{4}$ to his sister and the last apple he ate himself. How many apples did he originally have?
- (7) If you take half my age and add 7, you get my age 13 years ago. How old am I?
- (8) A horse and a mule, both heavily loaded, were going side by side. The horse complained of its heavy load. 'What are you complaining about?' replied the mule. 'If I take one sack off your back, my load will become twice as heavy as yours. But if you remove one sack from my back, your load will be the same as mine.' How many sacks was the horse carrying and how many sacks the mule?
- (9) Four brothers have 45 dollars. If the money of the first is increased by 2 dollars and the money of the second is decreased by 2 dollars, and the money of the third is doubled, and the money of the fourth is halved, then all of them will have the same amount of money. How much does each have?
- (10) There are two palm trees, one opposite the other on each side of a river. One is 30 cubits high, the other 20 cubits. The distance between the foot of each tree comes to 50 cubits. A bird is perched on the top of each tree. All of a sudden, the birds see a fish come to the surface of the river between the palm trees. They dive at the same time and reach the fish at the same time. Find the distance between the foot of the taller tree and the fish.

