

Math 5e, Homework 25

Due April 6

Instructions: Some of the problems we solved in class, and some are new. Please try to solve all problems, do your best, and show your work. **Write on separate sheets of paper, not between the lines of this handout!**

Geometry: Congruency

Two figures are called **congruent** if they have the same shape and size. We use the symbol \cong to denote congruent figures: to say that M_1 is congruent to M_2 , one writes $M_1 \cong M_2$.

The precise definition of the “same shape and size” depends on the figure. Most importantly, for triangles, it means that corresponding sides are equal and corresponding angles are equal: $\triangle ABC \cong \triangle A'B'C'$ is the same as: $AB = A'B'$, $BC = B'C'$, $AC = A'C'$,
 $\angle A = \angle A'$, $\angle B = \angle B'$, $\angle C = \angle C'$.

- Note that for triangles, the notation $\triangle ABC \cong \triangle A'B'C'$ tells that these two triangles are congruent and also shows which vertex of the first triangle corresponds to which vertex of the second one. For example, $\triangle ABC \cong \triangle PQR$ is not the same as $\triangle ABC \cong \triangle QPR$.

Congruent triangles

Rule 1 (Side-Side-Side rule). If $AB \cong A'B'$, $BC \cong B'C'$ and $AC \cong A'C'$ then $\triangle ABC \cong \triangle A'B'C'$.

This rule is commonly referred to as the **SSS** rule.

One can also define a triangle using three pieces of information, such as two sides and the angle between them. We will discuss it next time. This rule – and congruent triangles in general – are very useful for proving various properties of geometric figures.

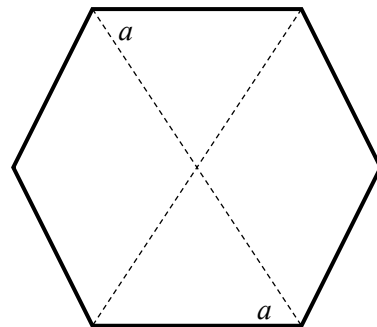
Parallelogram: A parallelogram is a quadrilateral in which opposite sides are parallel.

Sum of angles of an n -gon: is $(n - 2) \times 180$.

Homework

(Some problems were solved in class; review notes and please solve again)

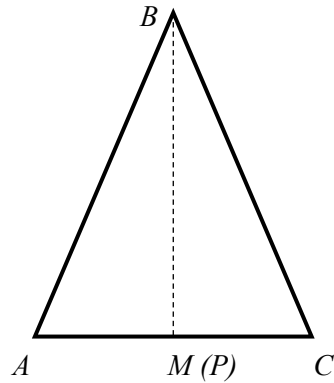
- An n -gon is called *regular* if all sides are equal and all angles are also equal.
 - How large is each angle in a regular hexagon (6-gon)?
 - Show that the opposite sides of a regular hexagon are parallel. (This is why this shape is used for nuts and bolts).
[Hint: show that each of the angles labeled by the letter a in the figure is equal to 60° , and then use the theorem about alternate interior angles.]



2. Let ABC be a triangle in which two sides are equal: $AB = BC$ (isosceles triangle). We proved in class that if M is the midpoint of the side AC , i.e. $AM = MC$, then

- ✓ triangles AMC and BMC are congruent.
- ✓ angles A and C are equal
- ✓ angle $AMC = 90$ degrees

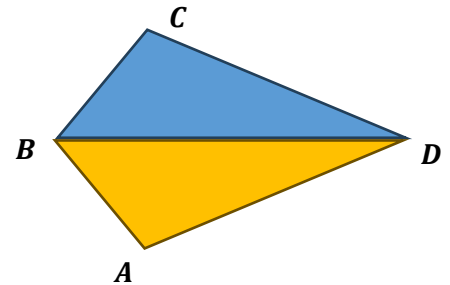
So, in an isosceles triangle, the median is also a height.



- a) Please review your notes and prove the above 3 points again!
- b) Can you prove the following: If in the triangle ΔABC $\angle A = \angle C$ and the point P of the side AC is such that BP is a height ($\angle BPC = 90^\circ$) then this triangle is isosceles?

3. For the sides of the quadrilateral $ABCD$ is given that $AB = BC$ and $AD = CD$. Show (prove) that:

- a) $\Delta ABD \cong \Delta CBD$
- b) BD splits the angles $\angle ABC$ and $\angle ADC$ into two equal parts (an angle bisector)



4. Simplify the expressions:

- (a) $x - (1 + 5x) =$
- (b) $2x - (3x^2 + x - 1) + (2 + 2x - x^2) =$
- (c) $3x \cdot (-2xy) =$
- (d) $(y - 5)(y - 1) - (y + 2)(y - 3) =$
- (e) $3(x - 1)^2 - 3x(x - 5) =$