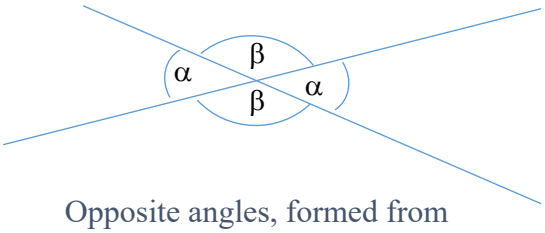
Math 5e, Homework 24

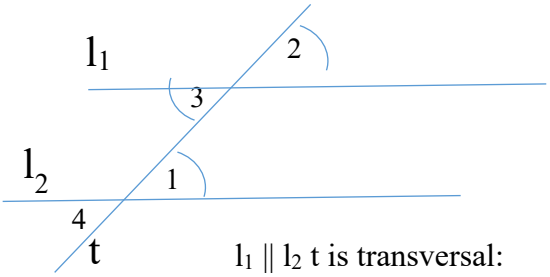
Due March 30

Instructions: Some of the problems we solved in class, and some are new. Please try to solve all problems, do your best, and show your work. Write on separate sheets of paper, not between the lines of this handout!

Geometry: Parallel lines, external angles, parallelograms



Opposite angles, formed from crossing straight lines, are equal.



$l_1 \parallel l_2$ t is transversal:
$\angle 1 = \angle 2 = \angle 3$

$\angle \alpha = \angle\alpha$ – opposite

$\angle\alpha + \angle\beta = 180^0$ – on a straight line,

Or complementary angles

$\angle 1 = \angle 3$ = alternate internal angles

$\angle 1 = \angle 2$ = corresponding angles

$\angle 4 = \angle 2$ = alternate exterior angles

Theorem 1: If two parallel lines (l1 and l2) are intersected by a third line (t), then the formed alternate interior angles are equal ($\angle 1 = \angle 3$).

Theorem 2: If two alternate interior angles formed when two lines crossed by a third are equal ($\angle 1 = \angle 3$), then the two lines (l1 and l2) are parallel.

Parallelogram: A parallelogram is a quadrilateral in which opposite sides are parallel. Parallelograms have several interesting properties, which we will study later.

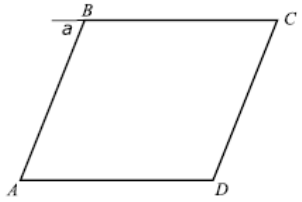
Sum of angles of an n-gon

Recall that the sum of angles of a triangle is 180^0. Since a quadrilateral can be cut into two triangles, the sum of the angles of a quadrilateral is $2\times180^0 = 360^0$. Similarly, for a pentagon, we get 3×180^0, and for an n-gon, the sum of angles is $(n - 2) \times 180^0$.

Homework

1. Is it true that any rectangle is also a parallelogram? Is it true that any parallelogram is a rectangle? Try to argue as carefully as you can.

2. Show that in a parallelogram, diagonally opposite angles are equal $\angle A = \angle C$, $\angle B = \angle D$ [Hint: see figure below, we did this in class, try to repeat the arguments]



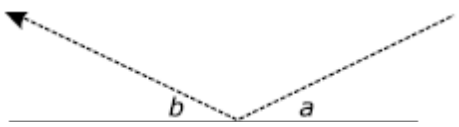
3. Show that the previous problem also works in the other direction: if in a quadrilateral, diagonally opposite angles are equal, $\measuredangle A = \measuredangle C$, $\measuredangle B = \measuredangle D$, then the quadrilateral must be a parallelogram.

4. Cut two identical paper triangles (the easiest way is to fold a sheet of paper in half and then cut). Can you put these two triangles together so that they form a parallelogram? Will your method always work? Why? Please explain.

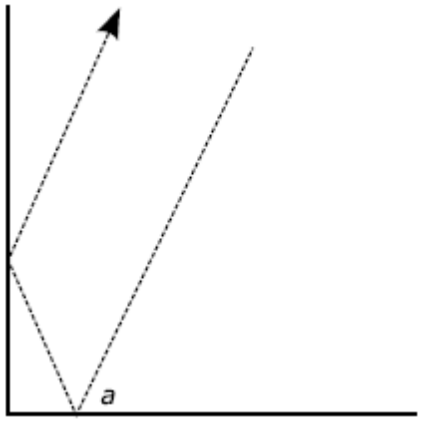
5. Antonia and Gabi took a 9-mile trip in a rowboat. There was only one pair of oars, so they took turns rowing (however, they didn't time how long each of them rowed, so it could happen that one had rowed longer than the other). Antonia could row at the speed of 3 miles per hour; Gabi could only do 2 miles per hour. It took them 3.5 hours to complete the trip. Can you find out how long each of them was rowing?

6. An n-gon is called **regular** if all sides are equal and all angles are also equal.
   (a) How large is each angle in a regular hexagon (6-gon)?
   (b) How large is each angle in a regular heptagon (7-gon)?

7. (Optional) In physics, the reflection law states that the angles formed by the incoming light ray and the reflected one with the surface of the mirror are equal: $\measuredangle a = \measuredangle b$



   Using this law, show that a corner made of two perpendicular mirrors will reflect any light ray exactly back: the reflected ray is parallel to the incoming one.



Hint: find the angle that each of these lines makes with the horizontal. Draw a horizontal line where the ray reflects from the vertical mirror.

This property – or rather, a similar property of corners in three dimensions — is widely used: reflecting road signs, car tail lights, and reflecting strips on clothing are all constructed from many small reflecting corners so that they reflect light from a car headlamp exactly back.