

Math 5e, Homework 20

Due February 16

Instructions: Some of the problems we solved in class, and some are new. Please try to solve all problems, do your best, and show your work. **Write on separate sheets of paper, not between the lines of this handout!**

Introduction to Probability

- We will be talking about “tests” or “events,” such as tossing a coin, rolling a die, or drawing a card. Each event can result in one of several possible outcomes: for example, rolling a die can give numbers 1 through 6. If there are n possible outcomes, and they are all equally likely, then the probability of obtaining any given one is exactly $1/n$. For example, the probability of rolling a 3 on a die is $1/6$.
- In most cases, we will be interested in the probability of something that can be obtained in more than one way. For example, if we ask what the probability of rolling an even number on a die is, then there are 3 ways to obtain it: by rolling 2, 4, or 6. Each of these outcomes has the probability $1/6$, so the probability of rolling one of them is $1/6 + 1/6 + 1/6 = 3/6 = 1/2$.

In general, the probability of obtaining one outcome from a certain collection of A possible outcomes, is given by

$$P(A) = \frac{\text{Number of outcomes giving } A}{\text{total number of possible outcomes}}$$

Addition Rule:

- Suppose one draws a card from a regular deck of 52 cards. What is the probability of drawing either a queen or a king? Since there are 4 queens and 4 kings, there are a total of 8 cards for this outcome. We can write: $P(Q \text{ or } K) = \frac{4+4}{52} = \frac{8}{52} = \frac{2}{13}$
- We can also write as follows: $P(Q \text{ or } K) = \frac{4+4}{52} = \frac{4}{52} + \frac{4}{52} = P(Q) + P(K)$

In general, we have the following rule: $P(A \text{ or } B) = P(A) + P(B)$

- Note: this is true if A and B cannot happen together. For example, there are 26 red cards in the deck, so the probability of drawing a red card is $\frac{26}{52} = \frac{1}{2}$. However, if we want a red card or a queen, then using the addition formula would give $\frac{26}{52} + \frac{4}{52} = \frac{30}{52}$, which is incorrect. In this way, we have counted the red queens twice. The correct answer is $\frac{28}{52}$: 26 red cards plus two black queens (no need to count the red queens – they have already been counted).

Complement Rule:

$$P(\text{not } A) = 1 - P(A)$$

- For example, the probability of drawing a queen from a deck of cards is $\frac{1}{13}$ thus, the probability of drawing something **other than** a queen is $1 - \frac{1}{13} = \frac{12}{13}$

Homework problems

1. Write each of the following expressions in the form $2^n 5^k$
 - (a) $\frac{2^2 5^8}{2^5 5^3}$
 - (b) $(2^3)^2 10^2 5^{-3}$
 - (c) $\frac{2^8 5^{-14}}{10^{-3}}$

2. In the roulette game, there are 37 slots, numbered 0 through 36. Half of the numbers 1–36 are red, the other half are black (zero has no color). What is the probability of hitting
 - (a) A number between 1 and 12
 - (b) An even number other than zero
 - (c) A red number or zero
 - (d) If you bet \$15 on odd numbers (i.e., you win if you roll an odd number), what is the probability of losing?

3. The standard card deck has 4 suits (hearts, diamonds, spades, and clubs); each suit has 13 different card values: 2 through 10, jack, queen, king, and ace. If you randomly draw one card, what is the probability of drawing
 - (a) The queen of spades
 - (b) A face card (i.e., jack, queen, or king)
 - (c) A black king
 - (d) Anything but the queen of hearts

4. Suppose I had drawn an ace from a deck of cards. What is the probability of randomly drawing a second ace from the remaining cards?

5. What is the probability that a randomly chosen person was born...
 - (a) in January? [Hint: January has 31 days]
 - (b) on February 5?
 - (c) on Sunday?
 - (d) On Sunday in January? [Hint: among all the people born in January, what fraction were born on Sunday?]

When solving this problem, you can ignore leap years and assume that birthdays are randomly distributed across all 365 days of the year, so each day is equally likely; in real life, this is not quite true.

6. Suppose we have a box of 500 candies of different colors and sizes. We know there are 100 large ones and 400 small ones; we also know there are 70 red ones, of which 11 are large. From this information, can you compute the probability that a randomly chosen candy will be either red or large (or both)?

7. When tossing a coin, it can land either with heads up or tails up (write H for heads, T for tails).
 - (a) If we toss a coin 3 times, what is the probability that all 3 will be heads? $P(\text{HHH}) = ?$
 - (b) If we toss a coin 3 times, what is the probability of obtaining this sequence of results: HTH? Is it more likely or less likely than getting all 3 heads? [Hint: write all possible combinations of 3 coins toss and count {HHH, HHT, ...}]