Math 5e, Spring 2026 Homework 17
due January 26

Instructions: Some of the problems we solved in class, and some are new. Please try to solve all problems, do your best, and show your work. Write on separate sheets of paper, not between the lines of this handout!

Square root.

The square root of a is a number whose square is equal to a. $\sqrt{a^2} = |a|$

Use the rules for powers to present $\sqrt{ab} = \sqrt{a}\sqrt{b}$.

$$(\sqrt{a})^2 = \sqrt{a}\sqrt{a} = \sqrt{aa} = \sqrt{a^2} = |a|$$

Theorem (Pythagorean theorem). In a right triangle with legs(sides) a, b and hypotenuse c, one has:

$$a^2 + b^2 = c^2, \text{ then } c = \sqrt{a^2 + b^2}$$

Irrational numbers

The square root of 2 ($\sqrt{2}$) is not a rational number, i.e., it cannot be written as a fraction.

Proof: Let us assume that $\sqrt{2} = \frac{p}{q}$, where p and q are some whole numbers and the fraction $\frac{p}{q}$ cannot be simplified further. We can write:

$$\left(\sqrt{2}\right)^2 = \left(\frac{p}{q}\right)^2$$

$$2 = \frac{p^2}{q^2}$$

$$2q^2 = p^2$$

Thus, p must be an even number and could be rewritten as $p = 2m$.

Substituting p: $\qquad\qquad\qquad\qquad 2q^2 = p^2 = 4m^2$

So that: $\qquad\qquad\qquad\qquad\qquad q^2 = 2m^2$

Thus, q must be an even number. This *contradicts* our initial assertion that $\frac{p}{q}$ could not be simplified further (at least each p, q could be reduced by one factor of 2 each). Therefore, we have proven by contradiction that $\sqrt{2}$ cannot be written as a rational number.

Homework problems

1. Simplify:

   a) $\left(\sqrt{17}\right)^2$

   b) $\left(\sqrt{13}\right)^4$

   c) $\left(\sqrt{11}\right)^3$

   d) $\left(\sqrt{3^4 3^6}\right)$

   e) $\left(\sqrt{2^4 3^5}\right)$

2. Can one cut a square with a side of 1m from the circle of diameter 1.4 m?

3. The side of an equilateral triangle is 1m. Find its height and area. Reminder that an equilateral triangle has all sides the same length.

4. Take a positive number $x < 100$, and using a calculator (or a computer), calculate the number $\frac{x}{2} + \frac{1}{x}$. Call the result x and repeat the same calculation with the new x. Do it 10 times. Then, take the result and square it. What did you get? Try to do the same thing, starting with a different number. What did you find? Is it surprising?

5. How many behemoths can one truck carry if the truck has a maximum load of 5 tonnes (5000 kg) and if each behemoth weighs 1500 kg? How many crocodiles can the same truck carry if each crocodile weighs 175 kg?

6. Calculate:

a) $(2^{-1})^2$; $\qquad$ $(3^{-2})^{-2}$; $\qquad$ $((-2)^{-1})^2$; $\qquad$ $\left(\left(3\frac{1}{7}\right)^0\right)^{-6}$

b) $\left(\frac{2(a+1)^3(a+1)^4}{3(a+1)^3}\right)^{-1}$

c) $2a^{-1} + 8(2a)^{-1} - 4\left(\frac{a^5}{a^4}\right)^{-1}$