

Math 5e, Fall 2025 Homework 13

Homework #13 is due December 22

Instructions: Some of the problems we solved in class, and some are new. Please try to solve all problems, do your best, and show your work. **Write on separate sheets of paper, not between the lines of this handout!**

Binary Numbers: Numbers represented by using only 0s and 1s.

Powers of 2

n	0	1	2	3	4	5	6	7	8	9
2^n	1	2	4	8	16	32	64	128	256	512

Numbers in decimal notation can be presented like this (same as converting a number to a decimal notation):
 $351 = 3 \times 100 + 5 \times 10 + 1 \times 1$

- Similarly, to convert a number into a binary, we need to represent it in powers of 2:
 $351 = 256 + 95 = 256 + 64 + 31 = 256 + 64 + 16 + 15 = 256 + 64 + 16 + 8 + 7 = 256 + 64 + 16 + 8 + 4 + 2 + 1$
 $351 = 1 \times 2^8 + 0 \times 2^7 + 1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 101011111$
- Or use the division by 2 method from class and write the remainders in reverse order.
- To convert binary numbers to decimal, we use the familiar rule: multiply each digit by its position value in base 2. For example:
 $1010 = 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 8 + 0 + 2 + 0 = 10$

Product formula: $(x - a)(x + a) = (x^2 - a^2)$

Homework problems

1. Solve the following equations:

a) $3(x - 1) - 4 = 3x + 8$
b) $\frac{1}{2}(x - 1) = -19$
c) $|2x| = 10$

2. Convert the decimal numbers to binary:

9, 12, 24, 38, 45

3. Convert the following binary numbers to decimal:

101, 1001, 10110, 11011, 10101

5. Can you find whole numbers a ; b such that $a^2 - b^2 = 17$? [Hint: Use the formula we talked about in class, and think about what $a - b$ and $a + b$ must be.]

4. Factor the following number into primes: $99^2 - 9^2$. [Hint: you do not have to compute this number, just the two factors as powers]

6. For the following problem, you need to know that the speed of light is about 300,000 km/sec, and one year is about 3×10^7 seconds. Answer using scientific notation (or a number \times power of 10).

- In astronomy, a common unit of distance is a light year: the distance light covers in one year. How many kilometers is it?
- Another common unit of distance in astronomy is the parsec, which is approximately 3×10^{13} km.
 - Can you compute how many parsecs are in one light-year?
 - How many parsecs is the distance between Earth and Sun (The distance is about 1.5×10^8 km)?
 - How many parsecs is the distance between Earth and the Andromeda Nebula (2,000,000,000,000,000 km)?