

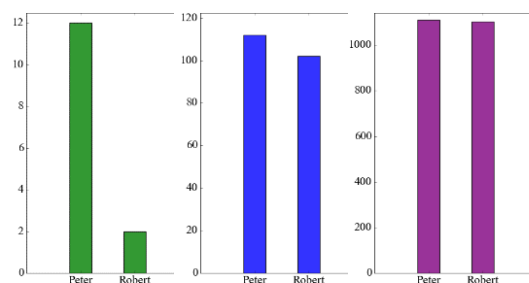
Ratio and proportions.

Peter has 10 dollars more than Robert. Is this a big difference? How we can compare the amount of money they have?

Take a look at the table

Peter	\$12	\$112	\$1112
Robert	\$2	\$102	\$1102

In all these cases the absolute difference is the same, but in the first case Peter has 6 times as much as Robert, in the last situation they both have almost the same amount of money. The ratios of the amount of Peter's money and Robert's money are.



$$\frac{12}{2}; \quad \frac{112}{102}; \quad \frac{1112}{1102};$$

The amount of money Peter and Robert have in the first case is 12 and 2 dollars and the ratio is $\frac{12}{2} = 6$, or 6:1, or 6 to 1.

Example1: (it's not a real recipe) The ratio of water and lemon juice in lemonade is 4 to 1. What does it mean? It means that for each part of lemon juice we need to add 4 parts of water, or the volume of lemon juice and water should have the same ratio as 1 to 4:

$$\frac{\text{volume of juice}}{\text{volume of water}} = \frac{1}{4}$$

How much of lemon juice and water we need to prepare 1 l. of lemonade? 1.5 l.?

Total number of volume "units" is $5 = 4 + 1$, for total volume, $\frac{1}{5}$ is juice, and $\frac{4}{5}$ is water.

We want to have sweet lemonade and we add sugar. The ratio of water, lemon juice and sugar is 4:1:0.5 (or it can be rephrased as 8:2:1). For each part of sugar, we will use 2 parts of lemon juice, and 8 parts of water.

We can write the ratio of two numbers in the several ways:

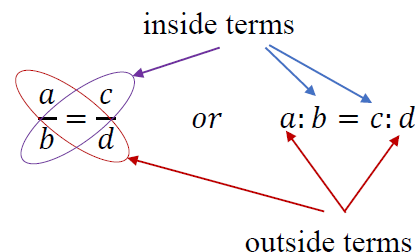
$$a \text{ to } b, \quad a:b, \quad \frac{a}{b}$$

Example 2:

To cook a raspberry jam according to recipe I need to combine three cups of berries and 2 cups of sugar, or for each 3 cups of raspberries go 2 cups of sugar; ratio of raspberries and sugar (in volume) is 3: 2. If I bought 27 cups of raspberries, how many cups of sugar do I need to put to my jam?

$$\frac{3}{2} = \frac{27}{x}$$

Two ratios which are equal form a proportion.
Proportions have several interesting features.



1. The products of inside and outside terms are equal.

$$\frac{a}{b} = \frac{c}{d} \Leftrightarrow a \cdot d = b \cdot c$$

2. Also, two inverse ratios are equal:

$$\frac{a}{b} = \frac{c}{d} \Leftrightarrow \frac{b}{a} = \frac{d}{c}$$

3. Two outside terms can be switched:

$$\frac{a}{b} = \frac{c}{d} \Leftrightarrow a \cdot d = b \cdot c \Leftrightarrow \frac{ad}{ab} = \frac{bc}{ab} \Leftrightarrow \frac{d}{c} = \frac{b}{a}$$

4. Two inside terms can be switched as well.

$$\frac{a}{b} = \frac{c}{d} \Leftrightarrow \frac{a}{c} = \frac{b}{d}$$

To finish the problem, we rewrite the cross multiplication and solve for x:

$$3 \cdot x = 2 \cdot 27$$

$$x = 2 \cdot 27 / 3 = 18$$

Homework:

- 1) The ratio of roses and hibiscuses in the garden is 9:11. What is the total number of flower bushes in the garden, if there are 99 rose bushes?
- 2) The ratio of cashews and walnuts in a nut mixture is 2:3, total weight of the mixture 150g. How many cashews and walnuts are in the pack of mixture?
- 3) To prepare a homemade dried fruits and nuts mix Mary took 6 parts of raisins, 5 parts of dried cranberries and 3 parts of walnuts. Cranberries and walnuts altogether weigh 2 kg 400 g. What was the weight of the mix that Mary prepared?
- 4) The whole family drank a full cup of coffee with milk, with Robert drinking a quarter of the milk and a sixth of all the coffee. How many people are in the family?

5) Solve the equations:

$$a. \frac{6x}{25} = \frac{0.4}{0.15}; \quad b. 1\frac{1}{9} : (0.8y) = \frac{1}{7} : 3.6; \quad c. \frac{1.25}{0.06} = \frac{z-6}{2.4}; \quad d. \frac{7}{2+t} = \frac{4.2}{t}$$

6) Evaluate

$$(2.5 - 0.75) \cdot \frac{4}{7} + \left(\left(3\frac{3}{8} - 2\frac{11}{12} \right) \cdot 1\frac{7}{9} + 2\frac{11}{12} \cdot 1\frac{7}{9} \right) : \left(3.5 : 2\frac{1}{3} \right)$$