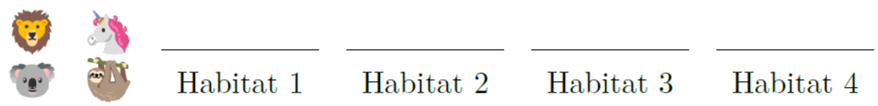
Math 5. Permutations

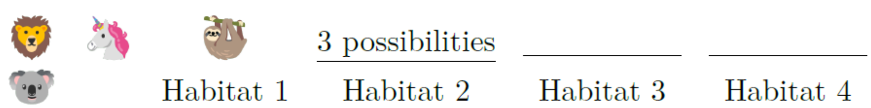
One common problem in Math is counting in how many different ways things can be ordered or arranged. In cases in which we do not have repeated objects and the order in which we arrange them matters, we call this a **permutation**.

Imagine, for example, that we are in charge of a natural reserve with four habitats and we have four different animals that we have to assign to each habitat: lion, unicorn, koala, sloth. We want to count in how many different ways we could arrange them.



An easy way to think about this is to first consider how many different possibilities you have for assigning an animal in the first habitat. Since there are four animals, we have four possibilities:

Suppose that we do not want the sloth to walk so much, so we assign it the first habitat. Then, we need to fill the second habitat, but now we will only have three possibilities, since there are three animals left:



As we keep making choices, the number of possible choices for the next case will decrease by one.

Finally, we only have one possibility for the missing animal:

To find the total number of possibilities in which we can arrange the animals, we **multiply** the different possibilities that we had for each habitat:

4 x 3 x 2 x 1 = 24

Imagine that instead of having 4 animals and 4 habitats, we have n animals and n habitats (where n could be any number). In this case, we have *n* possibilities for the first choice, *n-1* for the second, and so on, leaving a total of:

$$n \times (n-1) \times (n-2) \times \ldots \times 3 \times 2 \times 1 \text{ possibilities}$$

In general, if we are ordering k objects from a collection of n so that no repetitions are allowed, then this is referred to as a *permutation* of k objects from the collection of n, the number of ways to make such a selection of permutations is called ${}_nP_k$, and

$$\boldsymbol{{}_nP_k = \frac{n!}{(n-k)!}}$$

In particular, if we take $k = n$, it means that we are selecting one by one all n objects — so this gives the number of possible ways to order n objects:

$$\boldsymbol{{}_nP_n = n!} = n(n-1) \ldots \cdot 2 \cdot 1$$

We read $n!$ as "n factorial". By convention, $\mathbf{0}! = \mathbf{1}$, similar to the way that $x^0 = 1$.

Note that the number $n!$ grow very fast: $2! = 2, 3! = 6, 4! = 2 \cdot 3 \cdot 4 = 24, 5! = 120, 6! = 620$

For example:

$${}_6P_4 = 6 \cdot 5 \cdot 4 \cdot 3 = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = \frac{6!}{2!}$$

Homework

1. Apartment building has 12 apartments and a parking lot for 12 cars (each family has different car). How many ways are there to park these 12 cars?
2. Today there were only 4 cars at the parking lot. How many ways are there to park 4 cars on a 12-place parking lot?
3. A sly elementary school teacher decides to play favorites without telling anyone. If they have 15 students in their class, in how many ways can they choose a favorite student, a second favorite student, and a third favorite student?
4. Calculate:

   $\frac{6!}{3!}$, $6! - 3!$, ${}_7P_3$