

Math 5. Combinations

We need to choose a team (3) of students to go to math count competition out of 20. All students are equally good at math, we will choose randomly. We know how to calculate the number of permutations of 3 out of 20:

$$P(20, 3) = \frac{20!}{(20 - 3)!} = \frac{20!}{17!} = 20 \cdot 19 \cdot 18 = 6840$$

For each three students, say Kate, Mitchel, and Alice, we have 6 possible permutations between them, $3!$:

Kate	Mitchel	Alice
Kate	Alice	Mitchel
Mitchel	Alice	Kate
Mitchel	Kate	Alice
Alice	Mitchel	Kate
Alice	Kate	Mitchel

All these teams were calculated as different teams, but in reality, all six of them are the same.

So, we have to divide our final $P(3, 20)$ by $3!$

$$C(20, 3) = \frac{20!}{(20 - 3)! \cdot 3!} = \frac{6840}{6} = 1140$$

Now we have 1140 possible combinations (not permutations) to choose 3 student team from 20.

General formula for combination

$${}_nC_k = \binom{n}{k} = \frac{n!}{k!(n - k)!} = \frac{{}_nP_k}{k!}$$
$$C(n, m) = \frac{n!}{(n - m)! m!}$$

$m!$ is a number of permutations inside of group of m objects, we have to divide by that, when order does not matter.

Homework:

1) There are three cities in Wonderland A , B and C . There are 6 roads from city A to city B , and 4 roads from city B to city C . In how many ways can you travel from A to C ?

2) The sports club has 30 people, of which four people must be allocated to run the 1000 meters race. In how many ways can this be done?

3) In how many ways can a team of four be formed to participate in the 100m + 200m + 300m + 400m relay?

4) Compute $\binom{6}{3}$, $\binom{8}{5}$, $\binom{10}{0}$.