Math 5.

Rational numbers.

Positive rational number is a number which can be represented as a ratio of two natural numbers:

$$a = \frac{p}{q}; \qquad p, q \in N$$

As we know such number is also called a fraction, p in this fraction is a nominator and q is a denominator.

Basic property of fraction: nominator and denominator of the fraction can be multiplied by any non-zero number n, resulting the same fraction:

$$a = \frac{p}{q} = \frac{p \cdot n}{q \cdot n}$$

In the case that numbers p and q do not have common prime factors, the fraction $\frac{p}{q}$ is irreducible fraction. If $p < q$, the fraction is called "proper fraction", if $p > q$, the fraction is called "improper fraction".

If the denominator of fraction is a power of 10, this fraction can be represented as a finite decimal, for example,

$\frac{37}{100} = \frac{37}{10^2} = 0.37,$ $\frac{3}{10} = \frac{3}{10^1} = 0.3,$ $\frac{12437}{1000} = \frac{12437}{10^3} = 12{,}437$

$$10^n = (2 \cdot 5)^n = 2^n \cdot 5^n$$

$$\frac{2}{5} = \frac{2}{5^1} = \frac{2 \cdot 2^1}{5^1 \cdot 2^1} = \frac{4}{10} = 0.4$$

Therefore, any fraction, which denominator is represented by $2^n \cdot 5^m$ can be written in a form of finite decimal.

Periodic decimal

```
  0.71428571...
7|5.000
−0 0
  5 0
−4 9
   10
  − 7
    30
   −28
     20
    −14
      60
     −56
       40
      −35
        50
       −49
         10
          .....
```

If the denominator of the irreducible fraction has a factor different from 2 and 5, the fraction cannot be represented as a finite decimal. If we try to use the long division process, we will get an infinite periodic decimal.

At each step during this division, we will have a remainder. At some point during the process, we will see the remainder which occurred before. Process will start to repeat itself. On the example on the left, $\frac{5}{7}$, after 7, 1, 4, 2, 8, 5, remainder 7 appeared again, the fraction $\frac{5}{7}$ can be represented only as an infinite periodic decimal and should be written as $\frac{5}{7} = 0.\overline{714285}$. (Sometimes you can find the periodic infinite decimal written as $0.\overline{714285} = 0.(714285)$).

How can we represent the periodic decimal as a fraction?

Let's take a look on a few examples: $0.\overline{8}$, $2.35\overline{7}$, $0.\overline{0108}$.

$0.\overline{8}$.	$2.35\overline{7}$	$0.\overline{0108}$
$x = 0.\overline{8}$ $10x = 8.\overline{8}$ $10x - x = 8.\overline{8} - 0.\overline{8} = 8$ $9x = 8$ $x = \frac{8}{9}$	$x = 2.35\overline{7}$ $100x = 235.\overline{7}$ $1000x = 2357.\overline{7}$ $1000x - 100x = 2357.\overline{7} - 235.\overline{7}$ $= 2122$ $x = \frac{2122}{900} = \frac{1061}{450}$	$x = 0.\overline{0108}$ $10000x = 108.\overline{0108}$ $10000x - x = 108$ $x = \frac{108}{9999} = \frac{12}{1111}$

Homework.

1. Evaluate the following fractions:

   $a.\ \frac{2}{3} + 0.6;$ $b.\ 1\frac{1}{6} - 0.5;$ $c.\ 0.3 \cdot \frac{5}{9};$ $d.\ \frac{8}{11} : 0.4;$

2. Write as a fraction

   $a.\ 0.\overline{5},$ $b.\ 0.5,$ $c.\ 0.1\overline{2},$ $d.\ 0.\overline{12},$

3. The sorcerer used seaweed and mushrooms in a ratio of 5 to 2 when brewing a potion. How much seaweed does he need if there are only 450 grams of mushrooms?

4. The mass of a chicken egg is 80 g. Egg white makes up 55% of the total mass, and the yolk is 75% of the mass of egg white. Find the mass of the eggshell.

5. Evaluate:

$$\left(\frac{12 - \left(2\frac{1}{2} \cdot 3 + 9 : 5\right)}{\left(1\frac{7}{8} + 2\frac{11}{12}\right) : 3\frac{5}{6} + 0.1} \right)^3$$