

Rational numbers. Algebraic expression.

Expressions where variables, and/or numbers are added, subtracted, multiplied, and divided.

For example:

$$2a; \quad 3b + 2; \quad 3c^2 - 4xy^2$$

We can do a lot with algebraic expressions, even so we don't know exact values of variables. First, we always can combine like terms:

$$2x + 2y - 5 + 2x + 5y + 6 = 2x + 2x + 5y + 2y + 6 - 5 = 4x + 7y + 1$$

We can multiply an algebraic expression by a number or a variable:

$$3 \cdot (1 + 3y) = 3 \cdot 1 + 3 \cdot 3y = 3 + 9y$$

In this example the distributive property was used. Using the definition of multiplication we can write:

$$3 \cdot (1 + 3y) = (1 + 3y) + (1 + 3y) + (1 + 3y) = 3 + 3 \cdot y = 3 + 9y$$

Another example:

$$\begin{aligned} 5a(5 - 5x) &= \underbrace{(5 - 5x) + (5 - 5x) + \dots + (5 - 5x)}_{5a \text{ times}} = \underbrace{5 + 5 + \dots + 5}_{5a \text{ times}} - \underbrace{5x - 5x - \dots - 5x}_{5a \text{ times}} \\ &= \underbrace{5 + 5 + \dots + 5}_{5a \text{ times}} - \underbrace{5x - 5x - \dots - 5x}_{5a \text{ times}} = 5a \cdot 5 - 5a \cdot 5x = 25a - 25ax \end{aligned}$$

If we need to multiply two expressions

$$(a + 2) \cdot (a + 3)$$

We can use a substitution technic, we will substitute one of the expressions with a variable, for example, instead of $(a + 2)$ we can use u .

$$(a + 2) = u$$

And then we will multiply

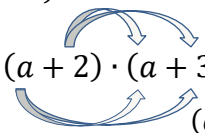
$$u \cdot (a + 3) = u \cdot a + 3u$$

We know, that actually u should not be there, $(a + 2)$ should.

$$u \cdot (a + 3) = (a + 2) \cdot a + 3(a + 2)$$

We know how to multiply an expression by a variable (or number):

$$(a + 2) \cdot a + 3(a + 2) = a \cdot a + 2a + 3a + 3 \cdot 2 = a^2 + 5a + 6$$

$$\begin{aligned} (a + 2) \cdot (a + 3) &= a \cdot a + 3a + 2a + 3 \cdot 2 = a^2 + 5a + 6 \\ (a + 2) \cdot (a + 3) &= a^2 + 5a + 6 \end{aligned}$$


Exercises.

1. Multiply.

a. $(a + 2)(a + 2);$ *b.* $(3 + y)(y + 4);$

c. $(3 + x)(3 - x);$ *d.* $(y - 2)(3 - y);$

e. $(2a + c)(a + ac);$ *f.* $(2d + 3l)(2d + 3l)$

2. Represent as fractions;

$1.23\overline{56},$

$5.4\overline{345}$

3. Represent as decimal:

$\frac{2}{5}; \quad \frac{2}{9}; \quad \frac{2}{8}; \quad \frac{2}{15}; \quad \frac{3}{16};$

4. Evaluate (use the properties of exponents):

$$\frac{3^{10} \cdot (3^2)^4}{(3^5)^3 \cdot 3}$$

5. Write the number 100 in binary and ternary (base of 3) systems.