Exponent.

Exponentiation is a mathematical operation, written as a^n, involving two numbers, the base *a* and the exponent *n*. When *n* is a positive integer, exponentiation corresponds to repeated multiplication of the base. In other words, a^n is the product of multiplying *n* bases:

$$a^n = \underbrace{a \cdot a \cdot a \ldots \cdot a}_{n\ times}$$

In that case, a^n is called the *n*-th power of *a*, or *a* raised to the power *n*.
The exponent indicates how many copies of the base are multiplied together.

Properties of exponent.
Based on the definition of the exponent, a few properties can be derived.

$$a^n \cdot a^m = \underbrace{a \cdot a \ldots \cdot a}_{n\ times} \cdot \underbrace{a \cdot a \ldots \cdot a}_{m\ times} = \underbrace{a \cdot a \cdot a \ldots \cdot a}_{n+m\ times} = a^{n+m}$$

$$(a^n)^m = \underbrace{a^n \cdot a^n \cdot \ldots \cdot a^n}_{m\ times} = \underbrace{\underbrace{a \cdot a \cdot \ldots \cdot a}_{n\ times} \cdot \ldots \cdot \underbrace{a \cdot a \cdot \ldots \cdot a}_{n\ times}}_{m\ times} = a^{n \cdot m}$$

If a number *a* in a power on *n* multiplied by the number *a* one more time, the total number of multiplied bases increased by 1:

$$a^n \cdot a = \underbrace{a \cdot a \cdot a \ldots \cdot a}_{n\ times} \cdot a = \underbrace{a \cdot a \cdot a \cdot a \ldots \cdot a}_{n+1\ times} = a^{n+1} = a^n \cdot a^1$$

In order to have the set of power properties consistent, any number in the first power is the number itself. In other words, $a^1 = a$ for any number *a*.

Also, a^n can be multiplied by 1:
$a^n = a^n \cdot 1 = a^{n+0} = a^n \cdot a^0$

In order to have the set of properties of exponent consistent, $a^0 = 1$ for any number *a,* but 0.
If there are two numbers *a* and *b*:

$$(a \cdot b)^n = \underbrace{(a \cdot b) \cdot \ldots \cdot (a \cdot b)}_{n\ times} = \underbrace{a \cdot \ldots \cdot a}_{n\ times} \cdot \underbrace{b \cdot \ldots \cdot b}_{n\ times}$$
$$= a^n \cdot b^n$$

All these properties can be summarized:

1. $a^n = \underbrace{a \cdot a \cdot a \ldots \cdot a}_{n\ times}$
2. $a^n \cdot a^m = a^{n+m}$
3. $(a^n)^m = a^{n \cdot m}$
4. $a^1 = a$, for any a
5. $a^0 = 1$, for any $a \neq 0$
6. $(a \cdot b)^n = a^n \cdot b^n$

Positive and negative numbers:

- A positive number raised into any power will result a positive number.
- A negative number, raised in a power, represented by an even number is positive, represented by an odd number is negative.

If a number a in a power n is divided by the same number in a power m,

$$\frac{a^n}{a^m} = \frac{\underbrace{a \cdot a \cdot \ldots \cdot a}_{n\ times}}{\underbrace{a \cdot a \cdot \ldots \cdot a}_{m\ times}} = \left(\underbrace{a \cdot a \cdot \ldots \cdot a}_{n\ times}\right) : \left(\underbrace{a \cdot a \cdot \ldots \cdot a}_{m\ times}\right) = a^n : a^m = a^{n-m}$$

So, we can say that

$$a^{-1} = \frac{1}{a^1} = \frac{1}{a}; \qquad a^{-n} = \frac{1}{a^n};$$

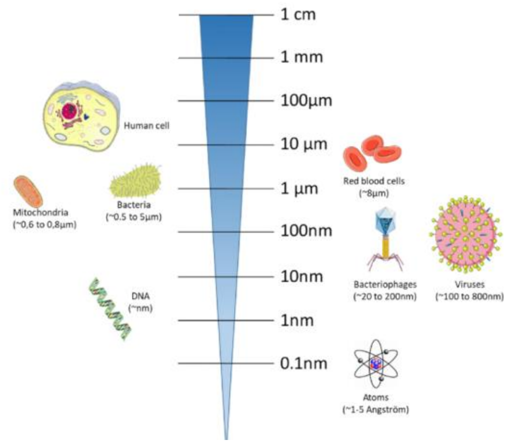
Let's see how our decimal system of writing numbers works when we use the concept of exponent:
$3456 = 1000 \cdot 3 + 100 \cdot 4 + 10 \cdot 5 + 1 \cdot 6 = 10^3 \cdot 3 + 10^2 \cdot 4 + 10^1 \cdot 5 + 10^0 \cdot 6$
The value of a place of a digit is defined by a power of 10 multiplied by the digit. Very large numbers can be written using this system, as well as very small numbers.

$$0.3 = \frac{1}{10} \cdot 3 = \frac{1}{10^1} = 10^{-1} \cdot 3$$

$$24.345 = 10^1 \cdot 2 + 10^0 \cdot 4 + 10^{-1} \cdot 3 + 10^{-2} \cdot 4 + 10^{-3} \cdot 5$$

Scientists work with very large and very small things, from galaxies to viruses. They need to be able to write numbers, describing the object of interest, for example the distance between two galaxies or the diameter of a virus.

A special way to write numbers is called scientific notation, it’s used a lot in science for describing various objects, big and small. Let’s take a look on the small things, like bacteria and viruses.



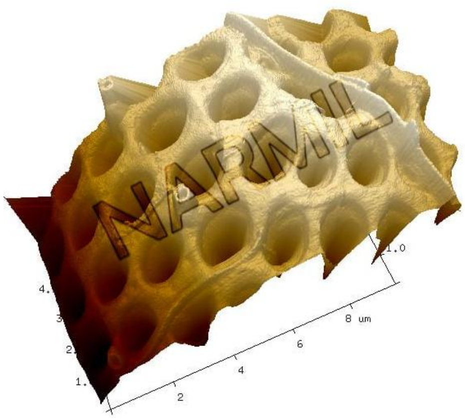
$1cm = 0.01m = \frac{1}{100}m = \frac{1}{10^2}m = 10^{-2}m$

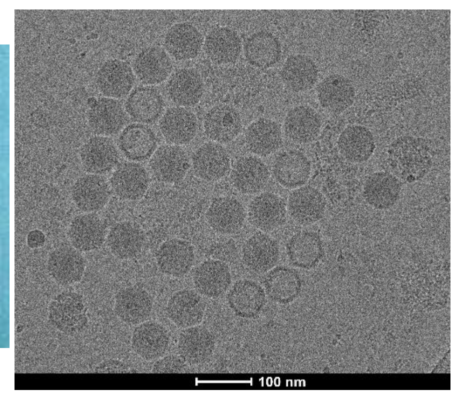
$1mm = 0.001m = \frac{1}{1000}m = \frac{1}{10^3}m = 10^{-3}m$

$1\mu m = 10^{-6}m, \quad 1nm = 10^{-9}m$

Bactria are between $0.5 - 1.5\ \mu m$

$0.5\mu m = 0.5 \cdot 10^{-6}m = 5 \cdot 10^{7}m$





Prefix	Symbol for Prefix		Scientific Notation
exa	E	1 000 000 000 000 000 000	10^{18}
peta	P	1 000 000 000 000 000	10^{15}
tera	T	1 000 000 000 000	10^{12}
giga	G	1 000 000 000	10^{9}
mega	M	1 000 000	10^{6}
kilo	k	1 000	10^{3}
hecto	h	100	10^{2}
deka	da	10	10^{1}
----	--	1	10^{0}
deci	d	0.1	10^{-1}
centi	c	0.01	10^{-2}
milli	m	0.001	10^{-3}
micro	µ	0.000 001	10^{-6}
nano	n	0.000 000 001	10^{-9}
pico	p	0.000 000 000 001	10^{-12}
femto	f	0.000 000 000 000 001	10^{-15}
atto	a	0.000 000 000 000 000 001	10^{-18}

Homework:

1. Continue the sequence:

$a.\ 1, 4, 9, 16 \ldots$ $b. 1, 8, 27, \ldots$ $c.\ 1, 4, 8, 16 \ldots$ $d.\ 1, 3, 9, 27 \ldots$

2. Write the following products as exponents:

Example:

$-2 \cdot 2 \cdot 2 \cdot 2 = -2^4;$ $(-2) \cdot (-2) \cdot (-2) \cdot (-2) = (-2)^4$

$a.\ (-3) \cdot (-3) \cdot (-3) \cdot (-3);$ $b.\ (-5m)(-5m) \cdot 2n \cdot 2n \cdot 2n;$

$c.\ -3 \cdot 3 \cdot 3 \cdot 3;$ $d.\ -5m \cdot m \cdot 2n \cdot n \cdot n;$

$e.\ (ab) \cdot (ab) \cdot (ab) \cdot (ab) \cdot (ab) \cdot (ab);$

3. What digits should be put instead of * to get true equality? How many solutions does each problem have?

$a.\ (2\,*)^2 = **\,1;$ $b.\ (3\,*)^2 = ***\,6$ $c.\ (7\,*)^2 = ***\,5$ $d.\ (2\,*)^2 = **\,9$

4. Without doing calculations, prove that the following inequalities hold:

Example:

$39^2 < 2000$: $39 < 40,$ $39^2 < 40^2 = 1600;$ $1600 < 2000.$

$a.\ 29^2 < 1000;$ $b.\ 48^2 < 3000;$ $c.\ 42^2 > 1500;$ $d. 67^2 > 3500$

5. Write as a power:

Example:

$\frac{1}{2} = 2^{-1};$ $\frac{1}{4} = \frac{1}{2^2} = 2^{-2}$

$\frac{1}{3};$ $\frac{1}{25};$ $\frac{1}{27};$ $\frac{1}{125};$