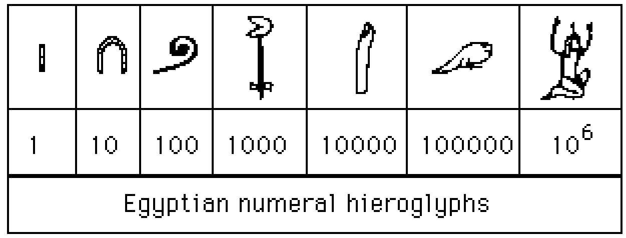
Math 5

**Numeral systems.**

Over the long centuries of human history, many different numeral systems have appeared in different cultures. The oldest systems weren't a place-valued system.
The good example of such system is the ancient Egyptian decimal numeral system.

It's decimal, but numbers can be written in an arbitrary order, but usually were red from right to left and/or from bottom to top.




1,333,330 in Egyptian hieroglyphs from the Edfu Temple (237–57 BCE) in Egypt.

The example on the right is from the Louvre and has the writing in columns. Notice the calf at the top of the picture is facing to the right. Thus, the hieroglyphs are read from top to bottom and, within each line, from right to left. The number pictured is composed of four lotus flowers (4000), six stacked coils of rope (600), two hobbles (20), and two tally marks (2), namely 4622. Since the number follows "calf," it translates as 4622 calves.

Another very well-known numeral system is roman; it was used for thousands of years and in some cases is still used now as well. It's a "decimal", 10 based system, but the symbols (letters) are used in an unusual way.

| Symbol | I | V | X | L | C | D | M |
|---|---|---|---|---|---|---|---|
| Value | 1 | 5 | 10 | 50 | 100 | 500 | 1000 |

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| I | II | III | IV | V | VI | VII | VIII | IX | X |

For example, 4 is one less than 5, so 4 can be written as IV. Same principle of subtractive notation is used for 9 -> IX, 40 and 90 -> XL and XC, 400 and 900 -> CD and CM
Some other examples:

- 29 = XX + IX = **XXIX**.
- 347 = CCC + XL + VII = **CCCXLVII**.
- 789 = DCC + LXXX + IX = **DCCLXXXIX**.
- 2,421 = MM + CD + XX + I = **MMCDXXI**

Any missing place (represented by a zero in the place-value equivalent) is omitted, as in Latin (and English) speech:

- 160 = C + LX = **CLX**
- 207 = CC + VII = **CCVII**
- 1,009 = M + IX = **MIX**
- 1,066 = M + LX + VI = **MLXVI**

Can non-decimal place-value system be created? For example, with base 5?
A numeral system's **base** is the total number of unique digits or symbols used to represent numbers in that system, including zero.
Let see, how we can create this kind of system (we use our normal digits).

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $Num_{10}$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| $Num_5$ | 1 | 2 | 3 | 4 | 10 | 11 | 12 | 13 | 14 | 20 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 30 | 31 | 32 | 33 | 34 | 40 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 41 | 42 | 43 | 44 | 100 | 101 | 102 | 103 | 104 | 105 |

We only have 5 digits (0, 1, 2, 3, 4), and 4 first "natural" numbers in such system will be represented as one-digit numbers.

We can transform the number decimal system to 5-base system (or any other system) by repeatedly dividing the number by the base, recording the remainders.

Let's convert 195 into 5-base system:

195 : 5 = 39R0

39 : 5 = 7R4

7 : 5 = 1R2

= $1240_5$

And vice versa, we can transform the number $2312_5$ from 5-base to decimal system:

**3 2 1 0**

$2312_5 = 5^3 \cdot 2 + 5^2 \cdot 3 + 5^1 \cdot 1 + 5^0 \cdot\ 2\ = 250 + 75 + 5 + 2 = 332$

It is a base in a power of the digit position (we go right to the left starting from zero) times the digit in this position.

There is another very important place-value system: binary system, base 2 system where only two digits exist; 0, and 1.

| $Num_{10}$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $Num_2$ | 1 | 10 | 11 | 100 | 101 | 110 | 111 | 1000 | 1001 | 1010 |

**Homework**:

1. Write number 67 in binary system.
2. Write the numbers, written in the binary system in decimal system:

   $a. 11011011_2;$ $b. 10001101_2,$

3. Write the number 245 and in 6-based place-value system. Remember, that in this system you will have only 0, 1, 2, 3. 4, and 5 as digits.
4. Write the number $403_6$ written in the 6-based place-value system (small number 6 shows that the number is not in decimal, but in 6-based system) in decimal system.
5. 7 wolves eat 7 sheep in 7 days. How many days will it take for 9 wolves to eat 9 sheep?