Math 5c, classwork 12.

Review of the place value systems.

The decimal system is a base-10 system for writing numbers.

$$\dots + 10^4 \cdot a + 10^3 \cdot b + 10^2 \cdot c + 10^1 \cdot d + 10^0 \cdot e + 10^{-1} \cdot f + 10^{-2} \cdot g + \cdots$$

Where $a, b, c, d \dots \in \{0,1,2,3,4,5,6,7,8,9\}$, or we can say they are digits.

A few examples of writing numbers in extended decimal form:

$$3245.567 = 1000 \cdot 3 + 100 \cdot 2 + 10 \cdot 4 + 1 \cdot 5 + \frac{1}{10} \cdot 5 + \frac{1}{100} \cdot 6 + \frac{1}{1000} \cdot 7 =$$

$$= 10^3 \cdot 3 + 10^2 \cdot 2 + 10^1 \cdot 4 + 10^0 \cdot 5 + 10^{-1} \cdot 5 + 10^{-2} \cdot 6 + 10^3 \cdot 7$$

$$274.98 = 100 \cdot 2 + 10 \cdot 7 + 1 \cdot 4 + \frac{1}{10} \cdot 9 + \frac{1}{100} \cdot 8$$
$$== 10^2 \cdot 2 + 10^1 \cdot 7 + 10^0 \cdot 4 + 10^{-1} \cdot 9 + 10^{-2} \cdot 8$$

Instead of 10, we can create a system of writing numbers based on any other number, such as 2, 3, 4, 5, and so on. Not all of them are convenient to use, but some have found their niche in mathematics.

Base 3 (ternary) system.

In the ternary system, each place value represents a power of 3.

3^0	1
3^1	3
3^2	9
3^3	27
3^4	81
3^5	243

To write numbers in the ternary system we only need three digits, 0, 1, and 2.

The value of the position on the written number is defined by the power of 3, similar to the decimal system. Confusion can arise because there is no digit 3 in the ternary system. The number 3 is represented as 10. The extended form of the number 1123 in the ternary system is:

$$102_3 = 10^2 \cdot 1 + 10^1 \cdot 1 + 10^0 \cdot 2 \ (in\ ternary\ system)$$

But in the decimal system we have the digit 3, and the written form of a ternary system number can be represented by its equivalent decimal value.

$$102_3 = (in\ decimalsystem)3^2 \cdot 1 + 3^1 \cdot 1 + 3^0 \cdot 2 = 9 + 3 + 2 = 14_{10}$$

$$2112_3 = (in\ decimalsystem)3^3 \cdot 2 + 3^2 \cdot 1 + 3^1 \cdot 1 + 3^0 \cdot 2 = 27 \cdot 2 + 9 \cdot 1 + 3 \cdot 1 + 1 \cdot 2$$
$$= 54 + 9 + 3 + 2 = 68_{10}$$

To transfer the number from decimal to ternary system one needs to write the number as a sum of powers of 3 multiplied by 0, 1, or 2. For example:

$$(in\ decimal\ system)\ 75 = 54 + 18 + 3 = 3^3 \cdot 2 + 3^2 \cdot 2 + 3^1 \cdot 1$$
$$= 2210_3\ (in\ ternary\ system)$$

Numeral systems can be created based on any number, like 4, 5, 8, 11, even 1111, but another widely used system is binary, based on 2 system. In this system there are only 2 digits, 0 and 1. Number 2 is represented as 10.

To write number 110101_b (also can be written as 110101_2) in decimal system:

$$2^5 \cdot 1 + 2^4 \cdot 1 + 2^3 \cdot 0 + 2^2 \cdot 1 + 2^1 \cdot 1 + 2^0 \cdot 1 = 32 + 16 + 0 + 4 + 0 + 1 = 53$$

Based on the understanding of the concept, we can create an algorithm to easily do the representation number in decimal system to any other system.

Foe example, let's take a look to the previous example of $75 \rightarrow 2210_3$ and $53 \rightarrow 110101$

$75:3 = 25R0$	$53:2 = 26R1$
$25:3 = 8R1$	$26:2 = 13R0$
$8:3 = 2R2$	$13:2 = 6R1$
$2:3 = R2$	$6:2 = 3R0$
	$3:2 = 1R1$
	$1:2 = 0R1$

If the remainders are red from the bottom up, the numbers written in ternary or binary will be presented.

Exercises:

1. Write in extended (decimal) form numbers
   $a.\ 23.32;\quad b.\ 213.2;\quad c.\ 7.32145;$

2. Write number 221201_3 in 10-based (decimal) system.
3. Write the number 212 in ternary system.

4. Write the number 101101_b in decimal system.
5. Write the number 147 in binary system.
6. Do the addition of numbers:
   $a.\ 21321_4 + 3212_4;$ $b. 1012_3 + 2101_3;$ $c.\ 101101_2 + 110101_2;$

7. Do the subtraction:
   $a.\ 21321_4 - 3212_4;$ $b. 2012_3 - 1101_3;$ $c.\ 111110_2 - 110101_2;$

Geometry.

A **definition** is a statement of the meaning of a something (term, word, another statement).

Desk *noun*

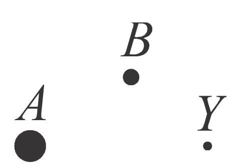
noun: **desk**; plural noun: **desks**

1. a piece of furniture with a flat or sloped surface and typically with drawers, at which one can read, write, or do other work.

- Music
  a position in an orchestra at which two players share a music stand.
  "an extra desk of first and second violins"
- a counter in a hotel, bank, or airport at which a customer may check in or obtain information.
  "the reception desk"

In mathematics everything (mmm,,,, almost everything) should be very well defined. In our real life, it is also very useful and convenient to agree about terms and concepts, to give them a definition, before starting using them just to be sure that everybody knows what they are talking about. Now we move to geometry.

Can we give a definition to a point? Can we clearly define what a point is? What a line is? What a plane is? Mathematicians decided do not define terms "point", "straight line", and "plane" and to rely upon intuitive understanding of these terms.

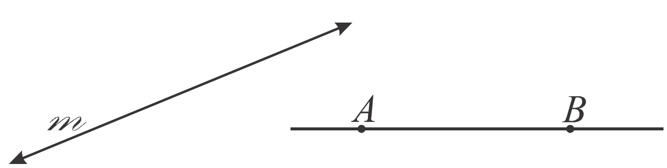


Point (an undefined term).

In geometry, a point has no dimension (actual size), point is an exact location in space. Although we represent a point with a dot, the point has no length, width, or thickness. Our dot can be very tiny or very large and it still represents a point. A point is usually named with a capital letter.

Line (an undefined term).

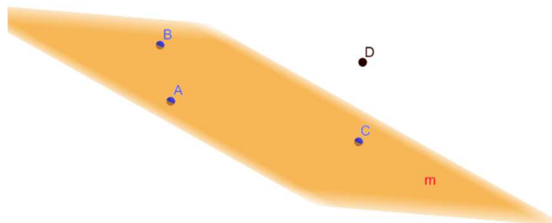
In geometry, a line has no thickness but its length extends in one dimension and goes on forever in both directions. Unless otherwise stated a line is drawn as a straight



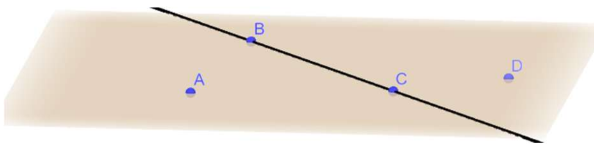
line with two arrowheads indicating that the line extends without end in both directions (or without them). A line is named by a single lowercase letter, m for example, or by any two points on the line, $\overleftrightarrow{AB}$ or AB.

Plane (an undefined term).

In geometry, a plane has no thickness but extends indefinitely in all directions. Planes are usually represented by a shape that looks like a parallelogram. Even though the diagram of a plane has edges, you must remember that the plane has no boundaries. A plane is named by a single letter (plane p) or by three non-collinear points (plane ABC).



Points can belong to the plane or can be outside of the plane. On a plane, points can belong to the straight line, or can be positioned on ether half-plane.



A set of all points of a straight line between two specific points. These points are called endpoints.

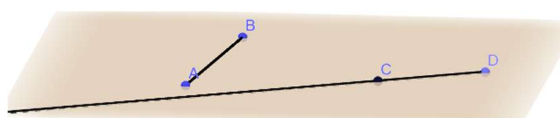
A ray is a part of a straight line consisting of a point (endpoint) and all points of a straight line at one side of an endpoint. Ray is named by endpoint and any other point, ray $\overrightarrow{AB}$ or AB (where A is an endpoint)

Draw a point on a sheet of paper.

- *How many straight lines can you draw through this single point?*

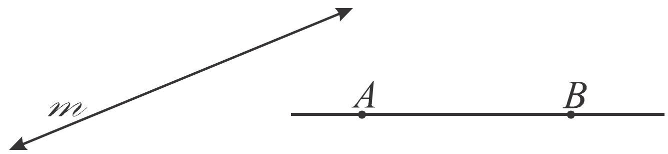
Draw another point.

- *How many straight lines can you draw through these two points?*



Through two points we always can draw a line (straight) and we can draw only one line.
If the line is drawn on plane, a point can be marked on this plane. This point can either lie on a line or not.

Draw a line and mark two points on the line. The part of the line between two points is a segment ([AB]).

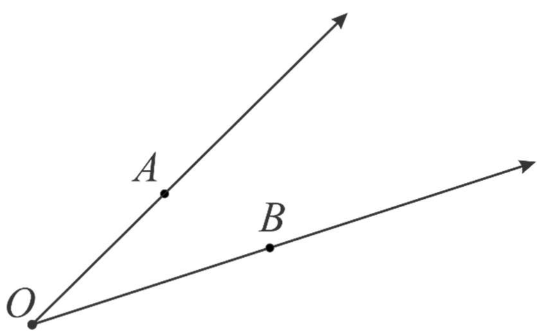


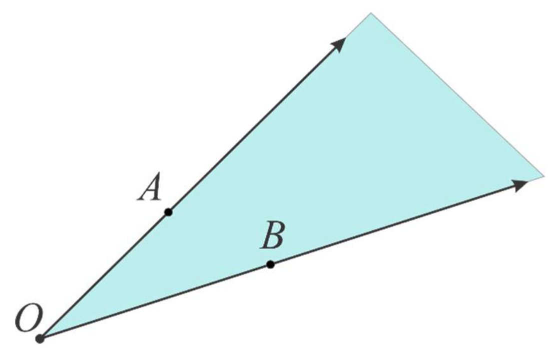
Part of a straight line on one side of a chosen point is a ray ($\overrightarrow{AB}$).

If we draw two rays from the same endpoint, we will get an angle.

- *Into how many parts does an angle divide a plane?*

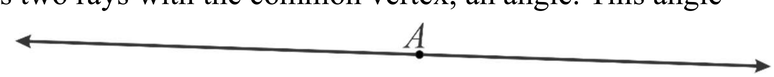
We can consider an angle to be two rays or two rays and the part of the plane they limit together. The difference only important when we look for the intersection of an angle and another geometrical figure.



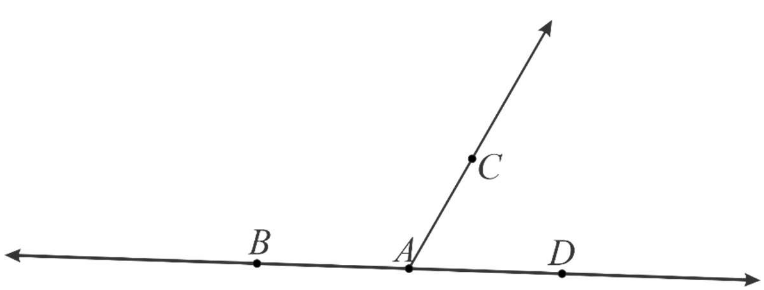


Angles notations are usually three capital letters with vertex letter in the middle or small Greek letter: ∠ABC, α.

If a point marked on a line, it produces two rays with the common vertex, an angle. This angle has it's own name: a straight angle.

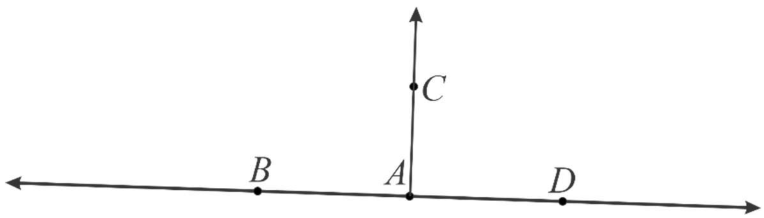


If another ray is coming from the vertex of a straight angle, we now have three angles, $\angle CAB, \angle CAD, \angle BAD$.



- *What can you say about these angles?*

Such angles we call supplementary angles.
There is only one angle which supplement itself to a straight angle. In this case supplementary angles are equal, and we call this angle a right angle. Measure of the straight angle is 180°, measure of the right angle is 90°



Exercises:

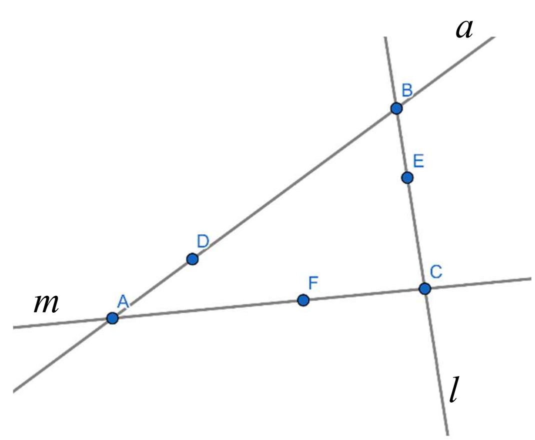
1. Draw a segment 2 cm long, 5 cm long, a square with the side 4 cm. (use ruler, pencil).

2. Draw two line segments AB and CD in such way that their intersect
a. by a point
b. by a segment
c. don't intersect at all.

3. Using a ruler draw a straight line, put on it 3 points, *A*, *B*, and *C* so that 2 rays are formed, *BC* and *BA*.

4. Draw two rays AB and CD in such way that their intersect
d. by a point
e. by a segment
f. by a ray
g. don't intersect at all.

5. Through which points does the lime *m* pass?
   Through which points does the lime *a* pass?
   What is the intersection of the lines *m* and *l*?



6. Mark 2 points. How many different lines can be drawn through these two points?

7. Mark three points. How many lines can be drawn through three points?
   Consider all possible solution.

8. Mark four points. (Any three points do not belongs to the same line).
   How many lines can be drawn through four points?
   10 points? 100 points?

9. Draw two segments so their intersection is
a. A point
b. A segment
c. No intersection

10. Draw two rays so that their intersection is
a. A point
b. A segment
c. A ray.

11. Let consider an angle as part of the lane along with two rays. Draw two angles so that their intersection with a line will be
a. A segment
b. A point
c. A ray
d. No intersection

12. Measure angles: