

Power of 2:

2^0	2^1	2^2	2^3	2^4	2^5	2^6
1	2	4	8	16	32	64

2^7	2^8	2^9	2^{10}	2^{11}	2^{12}	2^{13}
128	256	512	1024	2048	4096	8192

Power of 3

3^0	3^1	3^2	3^3	3^4	3^5	3^6
1	3	9	27	81	243	729

a	a^0	a^1	a^2	a^3	a^4	a^5
4	1	4	16	64	256	1024
5	1	5	25	125	625	3125
6	1	6	36	216	1296	7776
7	1	7	49	343	1402	16807

Let's write number 782 in 5-based system:

The highest power of 5 which is smaller than 782 is $5^4 = 625$, taken once, therefore

$$782 = 625 + 157$$

The highest power of 5, which is smaller than 157 is $5^3 = 125$, taken once:

$$782 = 625 + 157 = 625 + 125 + 32$$

The highest power of 5, which is smaller than 32 is $5^2 = 25$, taken once:

$$782 = 625 + 157 = 625 + 125 + 25 + 7 = 625 + 125 + 25 + 5 + 2$$

$$782 = 5^4 + 5^3 + 5^2 + 5^1 + 5^0 \cdot 2 = 11112_5$$

Another way to write a number in 5-based system:

$$782 \div 5 = 156 \text{ remainder } 2$$

$$156 \div 5 = 31 \text{ remainder } 1$$

$$31 \div 5 = 6 \text{ remainder } 1$$

$$6 \div 5 = 1 \text{ remainder } 1$$

$$1 \div 5 = 0 \text{ remainder } 1$$

If these remainders are read from the bottom up, the same result will appear. Can you explain why? This is the easiest technical way to transfer number from decimal to n-based system, but to understand why the answer is right one, we have to learn the concept.

Another example:

Write the number 893 in 4-base system.

The highest power of 4, which is smaller than 893 is $4^4 = 256$, taken three times:

$$893 = 256 \cdot 3 + 125$$

The highest power of 4, which is smaller than 125 is $4^3 = 64$, taken one time

$$893 = 256 \cdot 3 + 64 + 61$$

The highest power of 4, which is smaller than 61 is $4^2 = 16$, taken three times:

$$893 = 256 \cdot 3 + 64 + 16 \cdot 3 + 13 = 256 \cdot 3 + 64 + 16 \cdot 3 + 4^1 \cdot 3 + 1$$

$$893 = 4^4 \cdot 3 + 4^3 + 4^2 \cdot 3 + 4^1 \cdot 3 + 4^0 \cdot 1 = 31331_4$$

$$893 : 4 = 223 \text{ remainder } 1$$

$$223 : 4 = 55 \text{ remainder } 3$$

$$55 : 4 = 13 \text{ remainder } 3$$

$$13 : 4 = 3 \text{ remainder } 1$$

$$3 : 4 = 0 \text{ remainder } 3$$

From 4-based to decimal:

Write 2132_4

$$2132_4 = 4^3 \cdot 2 + 4^2 \cdot 1 + 4^1 \cdot 3 + 4^0 \cdot 2 = 64 \cdot 2 + 16 \cdot 1 + 4 \cdot 3 + 2 = 158$$

From 5-based to decimal:

$$3423_5 = 5^3 \cdot 3 + 5^2 \cdot 4 + 5^1 \cdot 2 + 5^0 \cdot 3 = 125 \cdot 3 + 25 \cdot 4 + 5 \cdot 2 + 3 = 375 + 100 + 10 + 3 = 488$$

Exercises:

1. Write the numbers 234_6 and 403_6 written in the 6-based place-value system (small number 6 shows that the number is not in decimal, but in 6-based system) in decimal system.
2. Write 673, 753 in base-6 system.
3. Do the addition:

$$2134_5 + 3141_5;$$

$$21112_4 + 3211_4;$$

4. Do the subtraction:

$$4334_5 - 3441_5;$$

$$31212_4 - 12231_4;$$

5. Binary to decimal:

$$100001_2, \quad 110011_2$$

6. Decimal to binary:

$$132, \quad 111$$

7. Do the addition and subtraction:

$$1101101_2 + 111101_2; \quad 11100111_2 - 110011_2;$$

8. Do the multiplication:

$$101_2 \cdot 110_2; \quad 12_4 \cdot 23_4$$

9. There is a balance scale with two pans and one weight of each of the following masses: 1 g, 3 g, 9 g, 27 g, and 81 g. How can you balance a 61 g mass placed on one pan of the scale?

10. Robert thought of a number not less than 1 and not more than 1000. Julia is allowed to ask only such questions to which Robert can answer “yes” or “no” (Robert always tells the truth). Can Julia determine the hidden number in 10 questions?

11. There is a bag of sugar, a scale and a weight of 1 g. Is it possible to measure 1 kg of sugar in 10 weights?

12. How can 127 one-dollar bills be distributed among seven wallets so that any amount from 1 to 127 rubles can be given without opening the wallets?