Classwork 14.

Irrational numbers

Rational number is a number which can be represented as a ratio of two integers:

$$a = \frac{p}{q}; \qquad p \in Z, and\ q \in N, \qquad (Z = \{\pm \cdots, \pm 1, 0\}, N = \{1, 2, \ldots\})$$

Rational numbers can be represented as infinite periodical decimals (in the case of denominators containing only powers of 2 and 5 the periodical bloc of such decimal is 0).
Numbers, which can't be express as a ratio (fraction) $\frac{p}{q}$ for any integers p and q are irrational numbers. Their decimal expansion is not finite, and not periodical.
Examples:
0.01001000100001000001…
0.123456789101112131415161718192021…
What side the square with the area of a m^2 does have? To solve this problem, we have to find the number, which gives us a as its square. In other words, we have to solve the equation

$$x^2 = a$$

This equation can be solved (has a real number solution) only if a is nonnegative (($a \geq 0$) number. It can be seen very easily;

If $x = 0, \quad x \cdot x = x^2 = a = 0,$

If $x > 0, \quad x \cdot x = x^2 = a > 0,$

If $x < 0, \quad x \cdot x = x^2 = a > 0,$

We can see that the square of any real number is a nonnegative number, or there is no such real number that has negative square.

Square root of a (real nonnegative) number a is a number, square of which is equal to a.

There are only 2 square roots from any positive number, they are equal by absolute value, but have opposite signs. The square root from 0 is 0, there is no any real square root from negative real number.

Examples:

1. Find square roots of 16: 4 and (-4), $4^2 = (-4)^2 = 16$

2. Numbers $\frac{1}{7}$ and $\left(-\frac{1}{7}\right)$ are square roots of $\frac{1}{49}$, because $\frac{1}{7}\cdot\frac{1}{7}=\left(-\frac{1}{7}\right)\cdot\left(-\frac{1}{7}\right)=\frac{1}{49}$
3. Numbers $\frac{5}{3}$ and $\left(-\frac{5}{3}\right)$ are square roots of $\frac{25}{9}$, because $\left(\frac{5}{3}\right)^2=\frac{5}{3}\cdot\frac{5}{3}=\left(-\frac{5}{3}\right)^2=\left(-\frac{5}{3}\right)\cdot\left(-\frac{5}{3}\right)=\frac{25}{9}$

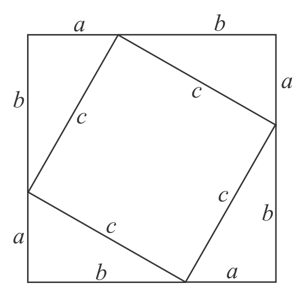
Arithmetic (principal) square root of a (real nonnegative) number a is a nonnegative number, square of which is equal to a.

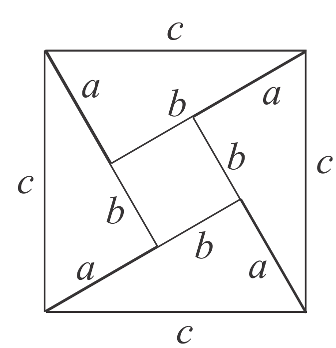
There is a special sign for the arithmetic square root of a number a: $\sqrt{a}$.
Examples;
1. $\sqrt{25}=5$, it means that arithmetic square root of 25 is 5, as a nonnegative number, square of which is 25. Square roots of 25 are 5 and (-5), or $\pm\sqrt{25}=\pm5$
2. Square roots of 121 are 11 and (-11), or $\pm\sqrt{121}=\pm11$
3. Square roots of 2 are $\pm\sqrt{2}$.
4. A few more:
   $\sqrt{0}=0$; $\sqrt{1}=1$; $\sqrt{4}=2$; $\sqrt{9}=3$; $\sqrt{16}=4$;
   $\sqrt{25}=5$; $\sqrt{\frac{1}{64}}=\frac{1}{8}$; $\sqrt{\frac{36}{25}}=\frac{6}{5}$

Pyphagorian theorem.

4 identical right triangles are arranged as shown on the picture. He area of the big square is $S=(a+b)\cdot(a+b)=(a+b)^2$, the are of the small square is $s=c^2$. The area of 4 triangles is $4\cdot\frac{1}{2}ab=2ab$. But also cab be represented as $S-s=2ab$

$$2ab=(a+b)\cdot(a+b)-c^2$$
$$=a^2+2ab+b^2-c^2$$
$$\Rightarrow \quad a^2+b^2=c^2$$

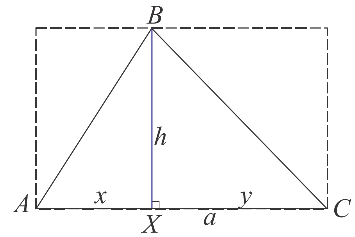




Area of a triangle

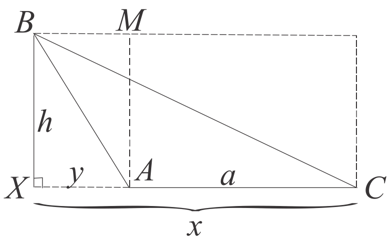
The area of a triangle is equal to half of the product of its altitude and the base, corresponding to this altitude.

For the acute triangle it is easy to see.



$$S_{rec} = h \cdot a = h \cdot (x + y) = hx + hy$$

$$S_{\Delta ABX} = \frac{1}{2}h \cdot x, \qquad S_{\Delta XBC} = \frac{1}{2}h \cdot y, \qquad S_{\Delta ABC} = S_{\Delta ABX} + S_{\Delta XBC}$$



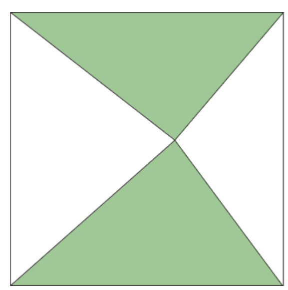
$$S_{\Delta ABC} = \frac{1}{2}h \cdot x + \frac{1}{2}h \cdot y = \frac{1}{2}h(x + y) = \frac{1}{2}h \times a$$

For an obtuse triangle it is not so obvious for the altitude drawn from the acute angle vertex. Can you come up with the idea how we can prove it for an obtuse triangle?

Area of trapezoid?

Exercises:

1. Which part of the square is shaded?



2. Prove that the value of the following expressions is a rational number.

   Example:

   $(\sqrt{3} - 1)(\sqrt{3} + 1) = \sqrt{3} \cdot \sqrt{3} + \sqrt{3} \cdot 1 - 1 \cdot \sqrt{3} - 1 = \sqrt{3} \cdot \sqrt{3} - 1 = (\sqrt{3})^2 - 1 = 3 - 1 = 2$

   $a.\ (\sqrt{2} - 1)(\sqrt{2} + 1);$ $\qquad b.\ (\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3})$

   $d.\ (\sqrt{2} + 1)^2 + (\sqrt{2} - 1)^2$ $\qquad d.\ (\sqrt{2} + 1)^2 + (\sqrt{2} - 1)^2$

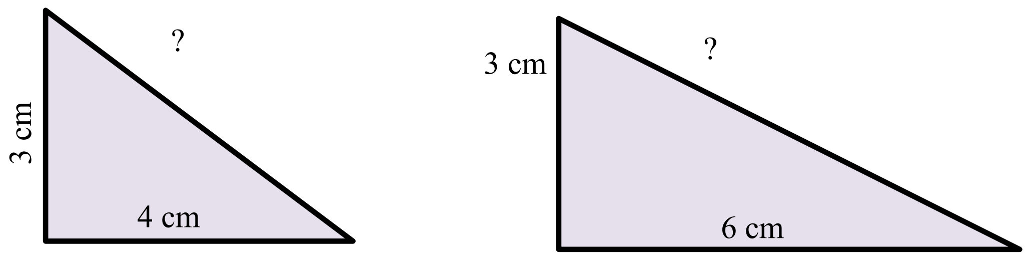
   $e.\ (\sqrt{7} - 2)^2 + 4\sqrt{7}$

3. Without using calculator compare:

$3 \ldots \sqrt{11}$ $\qquad$ $11 \ldots \sqrt{110}$ $\qquad$ $22 \ldots \sqrt{484}$

$5 \ldots \sqrt{20}$ $\qquad$ $17 \ldots \sqrt{299}$ $\qquad$ $35 \ldots \sqrt{1215}$

4. For the right triangles below find the missing side:



5. Compaire numbers:

$a. -1.\overline{7}\ and\ \frac{1}{7};$ $\qquad$ $b. -\frac{8}{9}\ \ and -\frac{9}{8};$ $\qquad$ $c.\ 0\ \ and -0.\overline{1234};$

$d.\ 4.\overline{5}\ and -5.\overline{4};$ $\qquad$ $e. -0.\overline{3}\ \ and -\frac{1}{3};$ $\qquad$ $f. -\frac{5}{99}\ \ and -0.\overline{04};$

$g. -\frac{10}{99}\ and -0.\overline{1};$ $\qquad$ $h.\ \frac{26}{99}\ \ and\ \ 0.26;$ $\qquad$ $i. -\frac{25}{99}\ \ and -0.25;$

6. Evaluate:

$a.\ \frac{2^5(2^3)^4}{2^{13}};$ $\qquad$ $b.\ \frac{3^7 \cdot 27}{(3^4)^3};$ $\qquad$ $c.\ \frac{4^6 \cdot 3^{10}}{6^{10}};$ $\qquad$ $d. \frac{24^4 \cdot 6^3}{48^3 \cdot 3^4};$