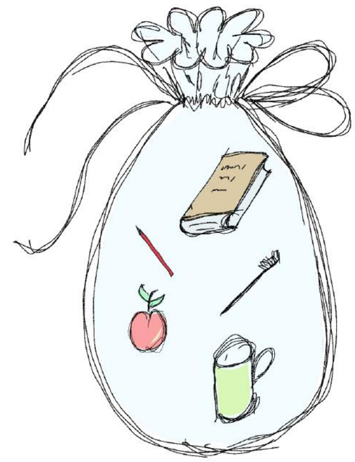
Math 5b. Classwork 11.
Sets and numbers.

I put a pencil, a book, a toothbrush, a coffee mug, and an apple into a bag. Do all these objects have something in common?



A set is a collection of objects that have something in common.

Can we call this collection of items a set? What is a common feature of all these objects? They are all in the bag, where I put them.

There are two ways of describing, or specifying, the members of a set. One way is by listing each member of the set, as we did with our set of things.

I can create, for example, a set of my favorite girls' names (F):

$\boldsymbol{F} = \{Mary,\ Kathrine, Sophia\}$.

The name of the set is usually indicated by a capital letter, in my case is F, list of members of the set is included in curved brackets. I can also create a set of all girls' names (N):

$\boldsymbol{N} = \{n|\ n = girl's\ name\}$

Of cause, I can't itemize all possible names, there are too many of them, we don't have enough space here for that, but I can describe the common feature of all members of the set – they are all girl's names. In the mathematical phrase above (1) I described the set, I call it N, which contains an unknown number of items, (we really don't know how many names exist), I use variable n to represent these items, girls' names. I show it by the phrase $n = girl's\ name$.

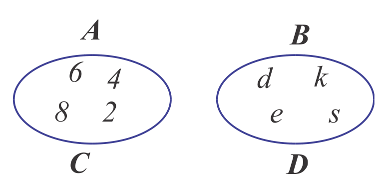
In our everyday life we use the concept of set quite often, we even have a special word for some sets, for example, the word "family" indicates a set of people, connected to each other, "class of 2020" – means all students who will graduate in 2020 and so on. Can you give more examples of such words and expressions?

When a set is created, about any object can be said that this object belongs to the set or not. For example, name "Emily" does not belong to the set F, number 2 also doesn't belong to this set. But "Emily" belongs to the set N, because it is a girl's name, number 2 is not a name.

Let's consider two examples of sets:

Sets A and B are created by listing their items explicitly:

A = {2, 4, 6, 8}, B = {d, e, s, k}.



Venn diagrams

Sets C and D are created by describing the rules according with which they were created:

C is the set of four first even natural numbers.

D is the set of letters of the word "desk".

If we look closer on our sets A and C, we can see that all elements of set A are the same as elements of set C (same goes for sets B and D).

$$\boldsymbol{A} = \boldsymbol{C} \text{ and } \boldsymbol{B} = \boldsymbol{D}$$

Two sets are equal if they contain exactly the same elements.

If a set A contains element '2', then we can tell that element '2' belongs to the set A. We have a special symbol to write it down in a shorter way: $2 \in A$, $105 \notin A$. (What does this statement mean?)

When a set is created, we can say about any possible element does it belong to the given set or not.

Let's define several sets:

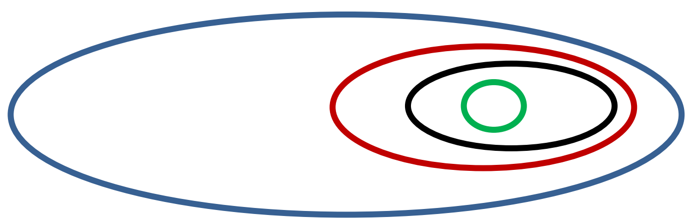
Set W will be the set of all words of the English language.

Set N will be the set of all nouns existing in the English language.

Set Z will be the set of all English nouns which have only 5 letters.

Set T={"table"}.

On a Venn diagram name all these sets:



If all elements of one set at the same time belong to another set then we can say that the first set is a subset of the second one. We have another special symbol to write this statement in a shorter way: $\subset$.

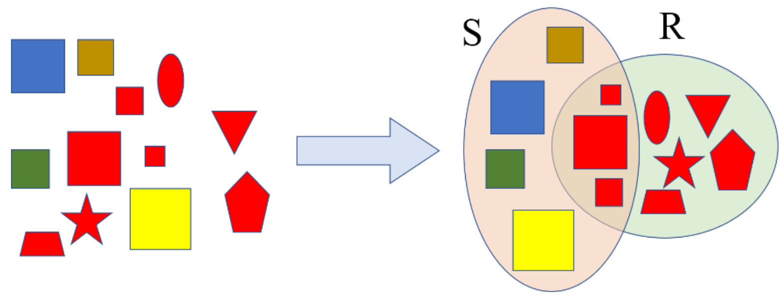
$T \subset Z \subset Y \subset W$

Problem:

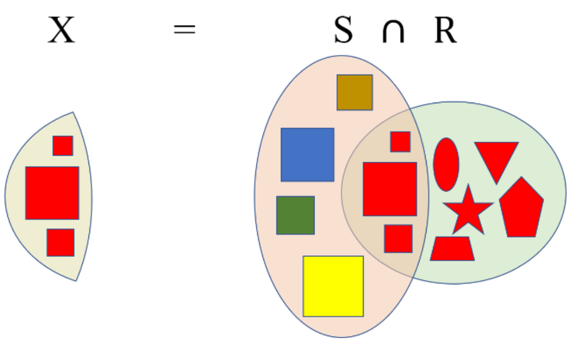
Draw Venn diagrams for the following sets:

Set A is the set of all cities of the United States. Set B is the set of all cities of the New-York state. Set C contains only one element, C = {Stony Brook}. Set D = {Paris(France), London(GB), Deli(India)}. Write the relationship between these sets.

When several sets are defined it can happen, that in accordance with all the rules we have implied, several objects can belong to several sets at the same time. For example, on a picture set S is a set of squares and a set R is a set of red shapes.



A few figures are squares and they are red, therefore they belong to both sets. Thus, we can describe a new set X containing elements that belong to the set S as well as to the set R. The new set was constructed by determining which members of two sets have the features of both sets. This statement also can be written down in a shorter version by using a special symbol ∩. Such set X is called an intersection of sets S and R.

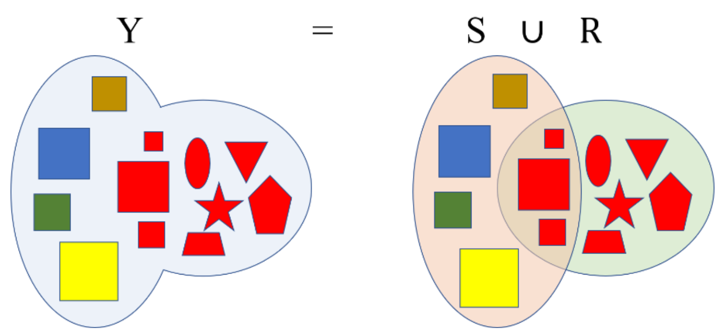


$\boldsymbol{X} = \boldsymbol{S} \cap \boldsymbol{R}$

Another new set can be created by combining all elements of either sets (in our case S and R). Using symbol ∪ we can easily write the sentence: Set Y contains all elements of set S and set R:

$\boldsymbol{Y} = \boldsymbol{S} \cup \boldsymbol{R}$

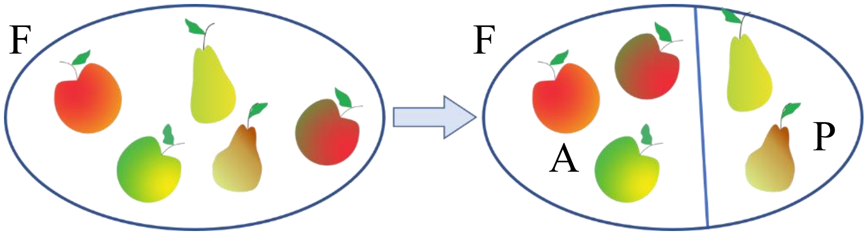
Such set Y is called a union of set S and R.



Set which does not have any element called an empty set (in math people use symbol ∅). For example, the set of polar bears living in the Antarctica is an empty set (there is no polar bears in Antarctica).

$\in$	element belongs to a set	$\not\subset$	one set is not a subset of another set
$\notin$	element does not belong to a set	$\cap$	intersection of two sets
$\subset$	one set is a subset of another set	$\cup$	union of two sets
$\varnothing$	empty set		

There is a basket with fruits on the table. It contains 3 apples and 2 pears. The set of fruits in the basket can be divided into two sets so, that these two sets do not intersect (apple can't be a pear at the same time), also in there is no any other elements in the set:



In this case we can say, we "classified" the set, we did the classification of the elements of the set.

$$A \subset F, \quad P \subset F, \quad A \cap P = \emptyset, \quad A \cup P = F$$

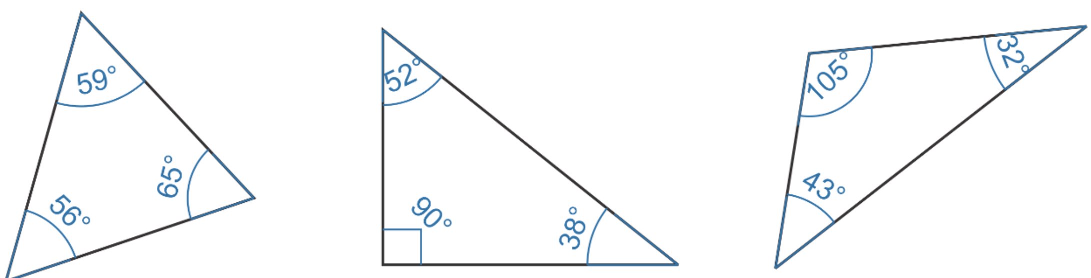
WE can now introduce the operation of addition and subtraction of sets.

$$F = A + P, \quad A = F - P, \quad P = F - A$$

The simplest polygon is a triangle. How we can classify triangles?

- By angles: triangles can be acute, obtuse, and wright triangles.

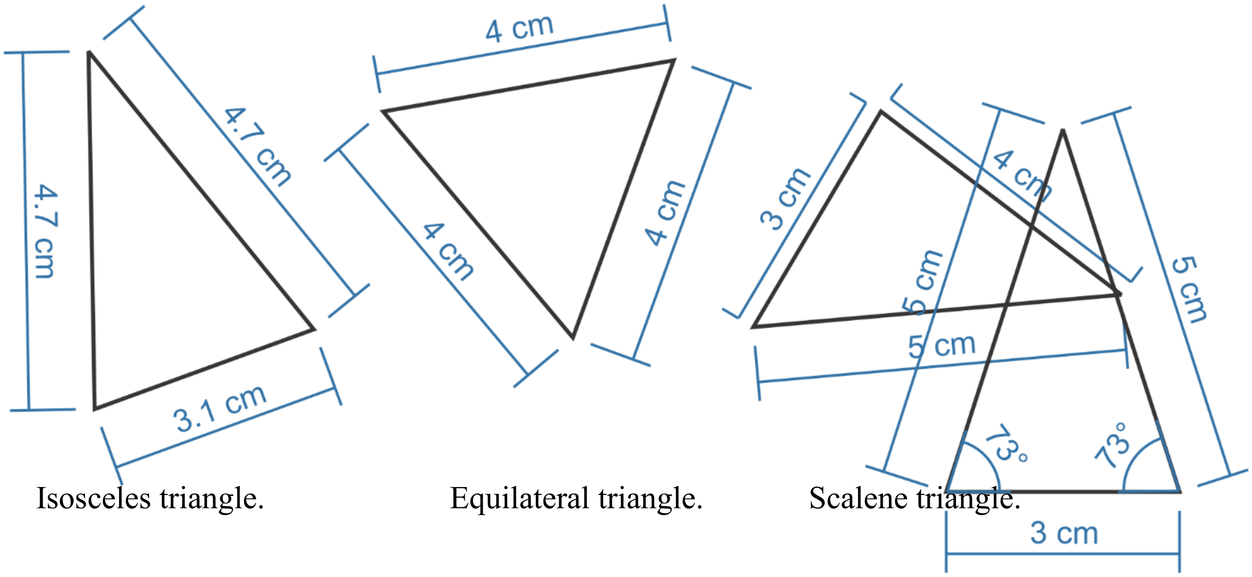
  Acute triangle has all three acute angles. (Acute angle is an angle less then wright angle).



Acute triangle. Wright triangle. Obtuse triangle.

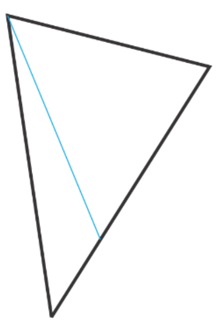
Obtuse triangle has one obtuse angle. Wright triangle has a wright angle.

- By sides: isosceles triangle, equilateral triangle, and scalene triangle. Isosceles triangle has two equal sides, equilateral triangle has all tree equal sides, and scalene triangle, with all three different sides.



Isosceles triangle. Equilateral triangle. Scalene triangle.

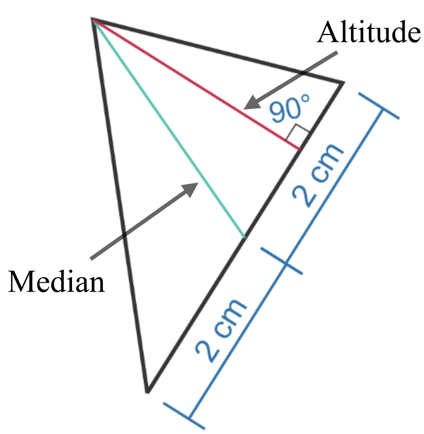
We know that bigger angle is opposite to the longer side. (This is a theorem, but we are not going to prove it for now). Base on this fact, it would be logical to suppose that in equilateral triangle all three angles are equal, and in isosceles triangle angels adjoined to the base are equal.

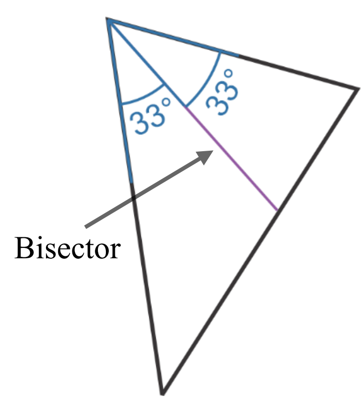


In each triangle we can connect a vertex with a point on the opposite side, like on the picture. Infinitely many segments can be drawn from any vertex to an opposite side, but there are three very special segments. From each vertex one can draw a segment to a midpoint of the opposite side, such segment is called a ***median***.

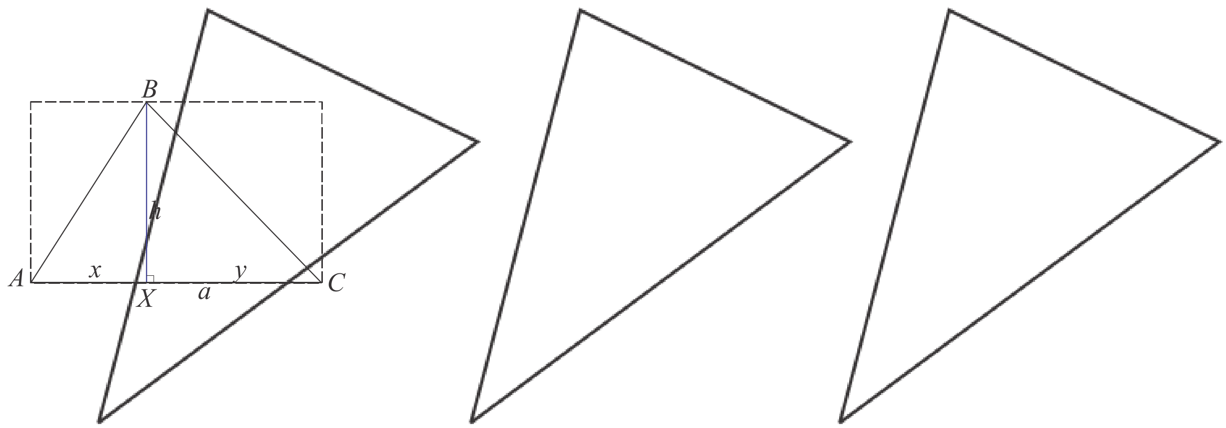
Also, from any vertex one can draw a perpendicular to an opposite side (or to continuation of the opposite side in the case of obtuse angle). This segment is called an ***altitude*** (or height).

And the last segment is an (angular) ***bisector***. It's a segment, drawn from any vertex of a triangle, in a way that the angel is divided into two equal angles.





Draw medians, altitudes and bisectors in the triangles:

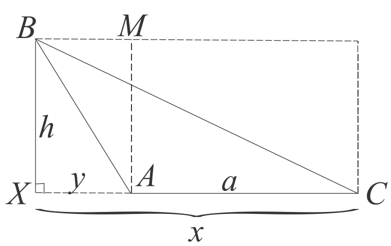


The area of a triangle is equal to half of the product of its altitude and the base, corresponding to this altitude.

For the acute triangle it is easy to see.

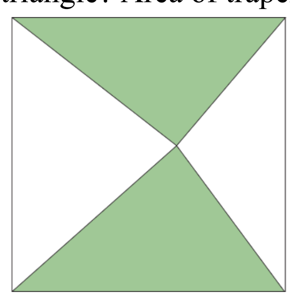
$$S_{rec} = h \cdot a = h \cdot (x + y) = hx + hy$$

$$S_{\Delta ABX} = \frac{1}{2} h \cdot x, \qquad S_{\Delta XBC} = \frac{1}{2} h \cdot y, \qquad S_{\Delta ABC} = S_{\Delta ABX} + S_{\Delta XBC}$$



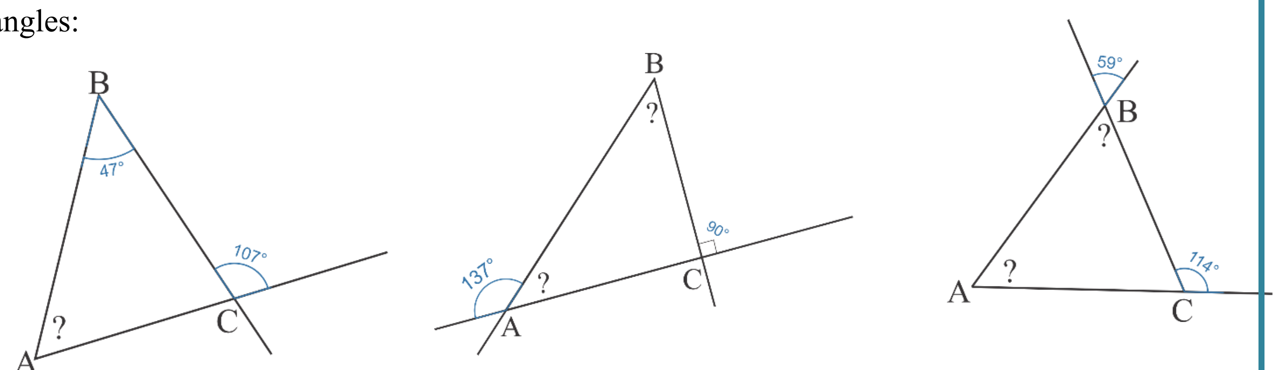
$$S_{\Delta ABC} = \frac{1}{2} h \cdot x + \frac{1}{2} h \cdot y = \frac{1}{2} h(x + y) = \frac{1}{2} h \times a$$

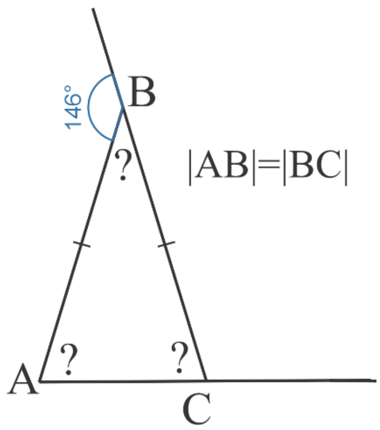
For an obtuse triangle it is not so obvious for the altitude drawn from the acute angle vertex. Can you come up with the idea how we can prove it for an obtuse triangle? Area of trapezoid?



Exercises:

1. Which part of the square is shaded?
2. Find angles:





3. Find the sum of

$$\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\frac{1}{4\cdot 5}+\cdots+\frac{1}{2024\cdot 2025}$$

4. Give examples of several members of the following sets:

   Example:

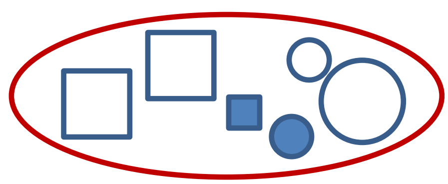
   $M = \{x|\ x = mammals\}$

   x can be a lion, a whale, a bat…

a. $K = \{y|\ y = letter\ of\ the\ english\ alfabet\}$
b. $M = \{x|\ x = flower\}$
c. $X = \{m|\ m = even\ number\}$
d. $P = \{k|\ k = color\}$

5. $A = \{2, 5, 0, 1\}$, $B = \{2, 0, 1\}$, $C = \{0, 2, 5, 1\}$, and $D = \{2, 0, 5, 4, 1\}$
   Which sets are equal?

6. How this set can be classified? Name the sets with capital letters, write all corresponding statements.



7. There are 21 students in a Math class. 10 students like apples and 15 students like pears. Show that there are some students, who like both apples and pears. Is it possible to determine if there any students who do not like apples and do not like pears? Explain your answer.

8. Assume, that each student likes at least one of the fruits. (This means that each student likes either apples, or pears, or both). How many students do like both pears and apples?

The same Math class (with 21 students) forms a soccer team and a basketball team. Every student signs up for at least one team: 12 students play only soccer; 2 students play both soccer and basketball; How many students play basketball only?

9. Students who participated in math coopetition had to solve 2 problems, one in algebra and another in geometry. Among 100 students 65 solved algebra problem, 45 solved geometry problem, 20 students solved both problems. How many students didn’t solve any problem at all?

10. On the diagrams of sets A, B, and C put 2 elements so that (just draw 2 points, or put any two letters).

a. each set contains 2 elements
b. set A contains 2 elements, set B contains also 2 elements, and set C contains 1 element.
c. set A contains 2 elements, sets B and C contain 1 element each
d. set A contains 2 elements, set B contains 1 element, and set C is an empty set
e. set A contains 2 elements, set B contains 2 elements, and set C is an empty set
f. each set contains 1 element