MATH 5: HANDOUT 21

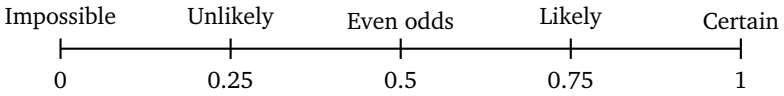
PROBABILITY I: INTRODUCTION (SUMMARY)

What is Probability?

Probability is the mathematics of chance. We study **experiments** with several possible **outcomes**.

Definition 1. The **probability** of an outcome is a number between 0 and 1 that measures how likely it is to occur.

- Probability 0 means the outcome is *impossible*.
- Probability 1 means the outcome is *certain*.



When all outcomes are **equally likely**:

$$P(A) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$$

Example: Rolling a die: $P(\text{even}) = \frac{3}{6} = \frac{1}{2}$ (three even numbers: 2, 4, 6 out of six)

Theoretical vs. Experimental Probability

Theoretical probability: Calculated using the formula above. What we expect *in theory*.
Experimental probability: Observed from actual trials:

$$P_{\text{exp}} = \frac{\text{times event occurred}}{\text{total trials}}$$

Law of Large Numbers: As you do more trials, experimental probability gets closer to theoretical probability.
Example: Flip a coin 100 times, get 47 heads. $P_{\text{exp}} = 47\%$, close to theoretical 50%.

The Addition Rule

For mutually exclusive events (cannot happen together):

$$P(A \text{ or } B) = P(A) + P(B)$$

Example: $P(\text{King or Queen}) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52}$

General Addition Rule (events may overlap):

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Example: $P(\text{red or Queen}) = \frac{26}{52} + \frac{4}{52} - \frac{2}{52} = \frac{28}{52}$

The Complement Rule

The probability that event A does *not* happen:

$$P(\text{not } A) = 1 - P(A)$$

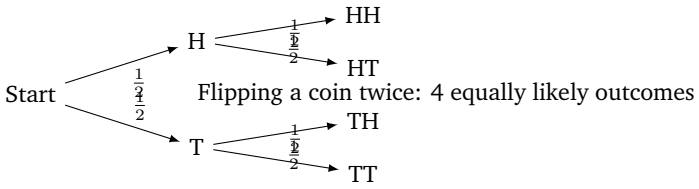
Example: $P(\text{not rolling a 6}) = 1 - \frac{1}{6} = \frac{5}{6}$

Complements of compound events: The complement of "red or Queen" is "**not** red **and not** a Queen" (i.e., black non-Queens). In general: *the complement of "A or B" is "not A and not B."*

Example: $P(\text{black non-Queen}) = 1 - P(\text{red or Queen}) = 1 - \frac{28}{52} = \frac{24}{52} = \frac{6}{13}$

Tree Diagrams

A **tree diagram** shows all possible outcomes. Each branch represents one outcome with its probability.



Key Formulas Summary

Concept	**Formula**
Basic probability	$P(A) = \dfrac{\text{favorable outcomes}}{\text{total outcomes}}$
Addition (mutually exclusive)	$P(A \text{ or } B) = P(A) + P(B)$
Addition (general)	$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
Complement	$P(\text{not } A) = 1 - P(A)$
Experimental probability	$P_{\text{exp}} = \dfrac{\text{times occurred}}{\text{total trials}}$

Common Mistakes to Avoid

- **Adding overlapping probabilities:** Don't use simple addition when events can happen together — subtract the overlap!
- **Probability out of range:** Probabilities must be between 0 and 1. If you get something else, check your work.
- **"At least one" vs. "exactly one":** These are different! "At least one" means one or more.
- **Assuming equal likelihood:** Not all outcomes are equally likely (e.g., a bent coin).

Homework

Problems marked with M *or unmarked are expected from every student. Problems marked with* H *are optional challenge problems.*

1. A jar contains 10 red candies, 7 blue candies, and 3 yellow candies. If you pick one candy at random, find:
   (a) The probability of getting a blue candy
   (b) The probability of getting a red or yellow candy
   (c) The probability of not getting a red candy
2. In the game of roulette, there are 37 slots numbered 0 through 36. Of numbers 1–36, half are red and half are black (zero has no color). Find the probability of:
   (a) Hitting a number between 1 and 12 (inclusive)
   (b) Hitting an even number other than zero

(c) Hitting a red number or zero

3. From a standard deck of 52 cards, find the probability of drawing:

   (a) The Queen of Spades

   (b) A black King

   (c) Anything except the Queen of Hearts

4. M You roll two dice (one red, one black).

   (a) How many total outcomes are possible?

   (b) What is the probability of rolling two 1s (snake eyes)?

   (c) What is the probability of rolling a 4 on the red die and a 6 on the black die?

5. M I draw a card from a deck and it turns out to be an Ace. Now I draw another card from the same deck (without replacing the first card). What is the probability that the second card is also an Ace?

6. What is the probability that a randomly chosen person was born:

   (a) In January? (Assume all months are equally likely)

   (b) On February 5? (Assume all 365 days are equally likely)

   (c) On a Sunday? (Assume all days of the week are equally likely)

7. M A box contains 500 candies of different colors and sizes. There are 100 large candies and 400 small ones. There are 70 red candies, of which 11 are large. What is the probability that a randomly chosen candy is either red or large (or both)?

8. M Using the same box of candies from Problem 7:

   (a) What is the complement of the event "red or large"? Describe it in words.

   (b) Find the probability of this complement by counting directly.

   (c) Find the probability of this complement using the complement rule and your answer to Problem 7. Verify that you get the same answer.

9. M You roll two dice. What is the probability that the product of the two numbers is a multiple of 3?

10. M If you toss a coin 5 times:

    (a) What is the probability that all 5 tosses are heads?

    (b) What is the probability of getting the exact sequence HHTHT?

    (c) Which is more likely: all heads, or the sequence HHTHT?

11. H You roll a die 3 times. What is the probability of rolling at least one 6?

12. H A family has two children. Given that at least one child is a boy, what is the probability that both children are boys? *(Hint: List all equally likely outcomes for two children, then exclude impossible cases.)*