# MATH 5: HANDOUT 20

## GEOMETRY VII: SIMILAR TRIANGLES (SUMMARY)

## Similar Figures

Two figures are **similar** (written $\sim$) if one can be obtained from the other by uniform scaling (stretching or shrinking equally in all directions).
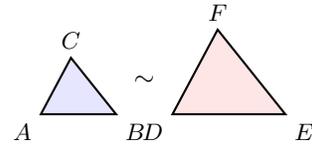
In similar figures:

- All corresponding **angles** are equal.

- All corresponding **sides** are proportional (in the same ratio).

If $\triangle ABC \sim \triangle DEF$ with **scale factor** $k$, then:

$$\angle A = \angle D, \quad \angle B = \angle E, \quad \angle C = \angle F$$

$$\frac{DE}{AB} = \frac{EF}{BC} = \frac{FD}{CA} = k$$

## AA Similarity Test

**AA Rule:** If two angles of one triangle equal two angles of another triangle, then the triangles are similar. (Since angles sum to $180°$, two equal angles guarantee the third is also equal.)

## Parallel Lines Create Similar Triangles

If line $DE \parallel BC$ in triangle $ABC$, then $\triangle ADE \sim \triangle ABC$.

The scale factor is $\dfrac{AD}{AB}$.

**Why?** Corresponding angles are equal (parallel lines cut by transversals), and they share angle $A$.
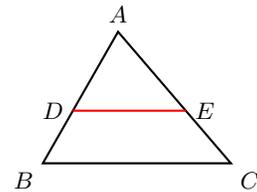
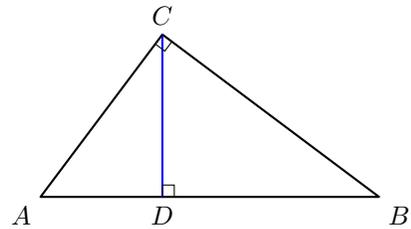## The Altitude to the Hypotenuse

In a right triangle $ABC$ with $\angle C = 90°$, let $CD$ be the altitude to hypotenuse $AB$. Three similar triangles are formed:

$$\triangle ABC \sim \triangle ACD \sim \triangle CBD$$

**Key formulas:**

$$AD = \frac{AC^2}{AB}, \quad BD = \frac{BC^2}{AB}, \quad CD = \sqrt{AD \cdot BD}$$
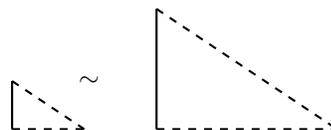
**Example:** If $AC = 6$, $BC = 8$, then $AB = 10$, so $AD = \frac{36}{10} = 3.6$, $BD = \frac{64}{10} = 6.4$, $CD = \sqrt{3.6 \times 6.4} = 4.8$.

## Using Similar Triangles: Shadows

When the sun casts shadows, objects and their shadows form similar triangles (same sun angle).

$$\frac{\text{object height}}{\text{shadow length}} = \text{same for all objects}$$

## Scale Factor and Area

When figures are similar with scale factor $k$:

- Lengths scale by $k$
- Areas scale by $k^2$

**Example:** Sides in ratio $2 : 3$ means $k = \frac{3}{2}$, so area ratio $= k^2 = \frac{9}{4}$.

**Circles:** Any two circles are similar with scale factor $k = \frac{r_2}{r_1}$.

**Pizza:** A 16" pizza has $2^2 = 4\times$ the area of an 8" pizza!

## Key Formulas Summary

| Concept | Formula/Rule |
|---|---|
| Similar triangles | $\dfrac{a'}{a} = \dfrac{b'}{b} = \dfrac{c'}{c} = k$ |
| AA Similarity | Two equal angles $\Rightarrow$ triangles are similar |
| Parallel line in triangle | Creates similar triangle with scale factor $\dfrac{\text{cut length}}{\text{full length}}$ |
| Altitude to hypotenuse | $AD = \dfrac{AC^2}{AB}, \quad CD = \sqrt{AD \cdot BD}$ |
| Area of similar figures | $\dfrac{\text{Area}_2}{\text{Area}_1} = k^2$ |

## Common Mistakes to Avoid

- **Similar $\neq$ Congruent.** Similar means same shape; congruent means same shape AND size.
- **Matching vertices correctly.** If $\triangle ABC \sim \triangle DEF$, then $A \leftrightarrow D$, $B \leftrightarrow E$, $C \leftrightarrow F$.
- **Area scales by $k^2$, not $k$.** If sides double, area quadruples!
- **AA is enough.** You don't need to check all three angles.

## Homework

1. Triangles $ABC$ and $PQR$ are similar. If $AB = 4$, $BC = 5$, $CA = 6$, and $PQ = 10$, find $QR$ and $RP$.

2. In the figure, $DE \parallel BC$. If $AD = 4$, $DB = 6$, and $BC = 15$, find $DE$.
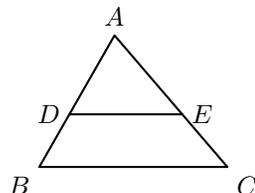


3. **M** In the story *The Musgrave Ritual*, Sherlock Holmes needed to find the length of a shadow cast by a tree that no longer existed. He knew the tree had been $64$ feet tall. To estimate the shadow, he set up a 6-foot rod and measured its shadow to be 9 feet. How long would the tree's shadow have been?

4. **M** In right triangle $ABC$ with $\angle C = 90°$, $AC = 4$ cm and $AB = 5$ cm. The altitude $CD$ is drawn to the hypotenuse.

   (a) Find $BC$.
   (b) Show that triangles $ABC$ and $ACD$ are similar.
   (c) Find $AD$ and $CD$.

5. **M** Two similar triangles have corresponding sides in ratio $2 : 5$.

   (a) If a side of the smaller triangle is 7, what is the corresponding side of the larger?
   (b) If the area of the larger triangle is $100$ cm², what is the area of the smaller?

6. **M** A 1.5-meter tall child stands 12 meters from a lamppost and casts a 3-meter shadow. How tall is the lamppost?

7. **H** In the figure, $ABCD$ is a parallelogram. $E$ is on $AB$ such that $AE : EB = 2 : 3$. Line $DE$ extended meets $CB$ extended at $F$. Find $BF : BC$.
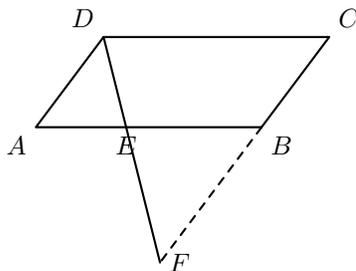


8. **H** Prove that the line connecting the midpoints of two sides of a triangle is parallel to the third side and equal to half its length. (This is called the **Midsegment Theorem**.)