

MATH 5: HANDOUT 18

GEOMETRY V: CIRCLES

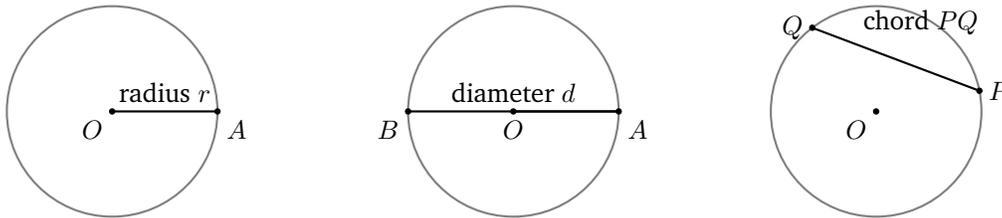
The Circle and Its Parts

A **circle** is the set of all points in a plane that are at the same distance from a fixed point called the **center**. That fixed distance is called the **radius**.

Radius. A **radius** (r) is any segment connecting the center of the circle to a point on the circle. All radii of a circle are equal.

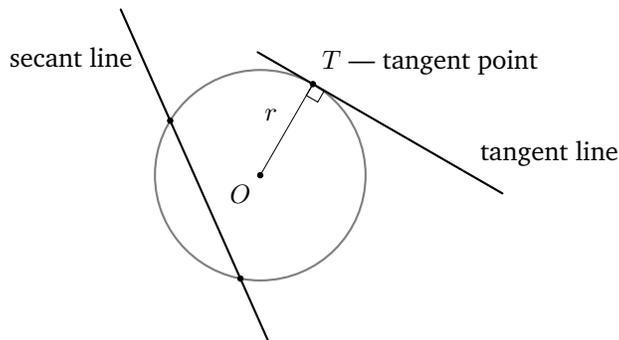
Diameter. A **diameter** (d) is a segment that passes through the center and has its endpoints on the circle. It equals twice the radius: $d = 2r$.

Chord. A **chord** is a segment whose endpoints lie on the circle. A diameter is the longest possible chord.



Secant. A **secant** is a line that intersects the circle in two points.

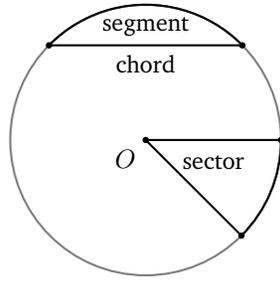
Tangent. A **tangent** is a line that touches the circle at exactly one point (called the **point of tangency**). A tangent is always *perpendicular to the radius* drawn to the point of tangency.



Arc. An **arc** is a part of the circle's circumference between two points.

Sector. A **sector** is the region enclosed by two radii and the arc between them—like a “slice of pizza.”

Segment. A **segment of a circle** is the region bounded by a chord and the arc it subtends—like a “slice of orange peel.”



Example 1. Summary of Circle Terms.

- **Center:** fixed point of the circle.
- **Radius:** distance from the center to any point on the circle.
- **Diameter:** chord through the center, equal to $2r$.
- **Chord:** segment with both endpoints on the circle.
- **Secant:** line that cuts the circle in two points.
- **Tangent:** line touching the circle at exactly one point, perpendicular to the radius.
- **Arc:** part of the circle between two points.
- **Sector:** region between two radii and the connecting arc.
- **Segment:** region between a chord and its arc.

In short: The circle is one of the most symmetric and important geometric figures. All of its properties depend on one simple idea — all points are the same distance from the center.

Quick Check

1. A circle has radius 7 cm. What is its diameter?
2. True or False: A diameter is a chord.
3. A chord passes through the center of a circle. What is it called?

Measuring a Circle — Circumference and π

When we measure straight lines, we can use rulers. But what about a circle? How can we measure how long its edge is?

The length all the way around a circle is called its **circumference**.

Discovering the pattern. Take a circular lid, cup, or wheel. If you roll it once along a straight line and mark where it starts and stops, you'll see the circle's rim has covered a certain distance. That distance is the same every time — it's the **circumference** of the circle.

Mathematicians long ago noticed something remarkable: if you divide the circumference by the diameter of any circle, you always get the same number!

$$\frac{\text{circumference}}{\text{diameter}} = \pi$$

No matter how big or small the circle is — a coin, a plate, or the Moon — this ratio is always the same constant number, called the Greek letter π (pronounced "pie").

What is π ? π is an infinite, non-repeating decimal:

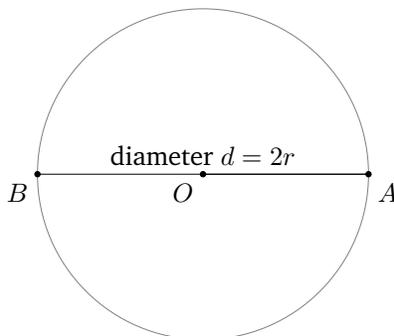
$$\pi = 3.1415926535 \dots$$

For most geometry problems, we can use $\pi \approx 3.14$ or the fraction $\frac{22}{7}$.

Formula for circumference. Because the ratio $C/d = \pi$, and $d = 2r$, we have:

$$C = \pi d = 2\pi r$$

circumference C (distance around the circle)



Circle area (for completeness). We'll explore this in detail later, but here is the famous formula:

$$A = \pi r^2$$

It means the area of a circle grows with the square of its radius.

Example 2. If the radius of a bicycle wheel is $r = 0.3$ m, then the circumference is

$$C = 2\pi r \approx 2 \times 3.14 \times 0.3 \approx 1.884 \text{ m.}$$

So one full turn of the wheel moves the bicycle forward about 1.9 m!

Summary:

- The circumference C is the distance around a circle.
- $C/d = \pi$, so $C = 2\pi r$.
- $\pi \approx 3.14$ is the same for all circles.
- The circle's area is $A = \pi r^2$.

Quick Check

4. Find the circumference of a circle with radius 5 (leave your answer in terms of π).
5. Find the area of a circle with radius 3 (leave your answer in terms of π).
6. A circle has diameter 10. What is its circumference?

Why Circles Matter — Rolling and the Cycloid

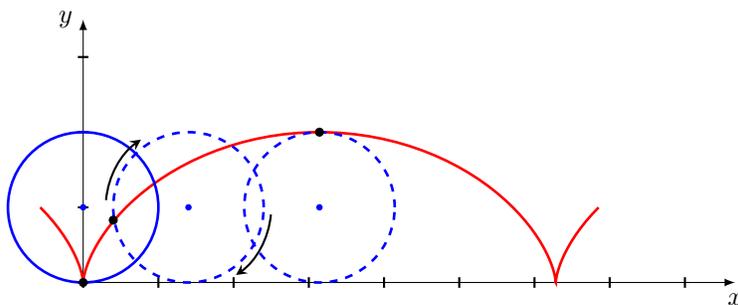
Circles are not just beautiful shapes — they are among the most practical forms in nature and human design. From planets and ripples in water to gears, wheels, and clocks, circular motion appears everywhere.

Why wheels are circular. Imagine trying to make a car with *square wheels*! As it rolls, the corners would keep lifting the body up and dropping it down again, creating a rough, bumpy ride. The center of a square wheel would move up and down as it turns.

A circular wheel, on the other hand, keeps its center at a constant height as it rolls. Every point on the rim stays the same distance from the center, so the rotation converts perfectly into smooth, forward motion. That's why all wheels — from bicycles to roller coasters — are circular.

The cycloid — a rolling circle's path. When a circle rolls along a straight line without slipping, a point on its rim traces a special curve called a **cycloid**. You can think of it as the path followed by a point of paint on the edge of a rolling wheel.

At first, the point touches the ground. As the wheel rolls, the point rises, moves forward, then dips back down — each loop corresponds to one full rotation of the circle.



You don't need the formula to understand the idea — just imagine the “bouncing point” on a rolling wheel. The cycloid combines both rotation and translation: the circle spins while moving forward.

Summary

- The circle is unique: when it rolls, its center moves in a straight line at constant height.
- Because of that, wheels, gears, and many moving parts are circular.
- A point on a rolling circle traces a curve called a *cycloid* — one of the most elegant shapes in mathematics.

Key Takeaways

- A circle is the set of all points at distance r (radius) from a fixed center.
- Diameter $= 2r$; the diameter is the longest chord.
- A tangent line touches the circle at exactly one point and is perpendicular to the radius at that point.
- Circumference $C = 2\pi r = \pi d$.
- Area $A = \pi r^2$.
- $\pi \approx 3.14159\dots$ is the same constant for all circles.

Common Mistakes

- **Confusing radius and diameter.** Remember: $\text{diameter} = 2 \times \text{radius}$. If given the diameter, divide by 2 to get the radius before using formulas.
- **Forgetting π in answers.** Unless asked to approximate, leave π in your answer (e.g., $C = 10\pi$, not $C = 31.4$).
- **Squaring the diameter instead of radius for area.** The area formula is πr^2 , not πd^2 . If you have the diameter, first find $r = d/2$.
- **Confusing circumference and area.** Circumference is a *length* (one-dimensional), while area is measured in *square units* (two-dimensional).

Classwork

- Fill in the blanks:
 - A circle with radius 6 has diameter _____.
 - A circle with diameter 20 has radius _____.
 - The longest chord in a circle is called the _____.
- Name the part of a circle described:
 - A line that touches the circle at exactly one point.
 - A segment from the center to a point on the circle.
 - The region between two radii and an arc.
- Find the circumference of each circle (leave answers in terms of π):
 - radius = 4
 - radius = 9
 - diameter = 14
- Find the area of each circle (leave answers in terms of π):
 - radius = 5
 - radius = 8
 - diameter = 12
- A circular pizza has diameter 16 inches.
 - What is the circumference of the pizza?
 - What is the area of the pizza?
- True or False:
 - Every diameter is a chord.
 - Every chord is a diameter.
 - A tangent line is perpendicular to the radius at the point of tangency.

Classwork Solutions

- Diameter = $2 \times 6 = 12$
 - Radius = $20 \div 2 = 10$
 - diameter
- Tangent
 - Radius
 - Sector
- $C = 2\pi(4) = 8\pi$
 - $C = 2\pi(9) = 18\pi$
 - $C = \pi(14) = 14\pi$
- $A = \pi(5)^2 = 25\pi$
 - $A = \pi(8)^2 = 64\pi$

(c) Radius = 6, so $A = \pi(6)^2 = 36\pi$

5. Radius = 8 inches.

(a) $C = 2\pi(8) = 16\pi$ inches ≈ 50.3 inches

(b) $A = \pi(8)^2 = 64\pi$ square inches ≈ 201 square inches

6. (a) True — a diameter is a chord that passes through the center.

(b) False — a chord can be shorter than the diameter.

(c) True — this is a key property of tangent lines.

Homework

1. Find the circumference and area of a circle with radius 7 (use $\pi \approx \frac{22}{7}$).
2. A circular garden has diameter 28 meters. Find its circumference and area.
3. **M** A wheel has circumference 62.8 cm. Find its radius (use $\pi \approx 3.14$).
4. A circular pond has area 154 square meters (use $\pi \approx \frac{22}{7}$). Find its radius.
5. **M** Compare the areas of two circles: one with radius 3 and one with radius 6. How many times larger is the area of the bigger circle?
6. **M** A semicircle (half circle) has diameter 10. Find:
 - (a) its area
 - (b) the perimeter of the semicircular region (the curved part plus the diameter)
7. The minute hand of a clock is 15 cm long. How far does its tip travel in one hour?
8. **M** A circular track has an outer radius of 50 m and an inner radius of 45 m. Find the area of the track (the ring-shaped region between the two circles).
9. **H** A rope is wrapped tightly around the Earth's equator (assume the Earth is a perfect sphere with circumference 40,000 km). If the rope is then lifted 1 meter above the ground all the way around, how much extra rope is needed?
10. **H** A quarter-circle (a 90° sector) has radius 8. Find its area and perimeter.

Quick Check Answers

1. Diameter = $2 \times 7 = 14$ cm.
2. True — a diameter is a chord that passes through the center.
3. A diameter.
4. $C = 2\pi(5) = 10\pi$.
5. $A = \pi(3)^2 = 9\pi$.
6. $C = \pi(10) = 10\pi$.