MATH 5: HANDOUT 4

SUBTRACTION AND WORD PROBLEMS WITH EQUATIONS

Subtraction

So far we have learned the basic rules of addition and multiplication with algebraic expressions. Today we add some rules involving **subtraction**. Subtraction must be handled carefully, because the minus sign applies to everything inside parentheses.

Rules of Subtraction

For any numbers a, b, c:

$$a - (b + c) = a - b - c$$
$$a - (b - c) = a - b + c$$
$$a(b - c) = ab - ac$$

Examples:

- $7 - (3 + 2) = 7 - 3 - 2 = 2$
- $10 - (6 - 4) = 10 - 6 + 4 = 8$
- $3(x - 5) = 3x - 15$

Solving Word Problems with Equations

Many real-life problems can be translated into algebraic equations. The key steps are:

1. Introduce a variable to represent the unknown.
2. Translate the conditions of the problem into an equation.
3. Solve the equation step by step.
4. Interpret the result in the context of the problem.

Example 1. An apple costs 9 cents and an orange 15 cents. Elena bought 20 pieces of fruit in total and paid $2.64. How many apples and how many oranges did she buy?
Solution. Let a = number of apples. Then the number of oranges is $20 - a$. The total cost is $9a + 15(20 - a)$ cents.

$$9a + 15(20 - a) = 264$$

Simplify:

$$9a + 300 - 15a = 264 \quad \Rightarrow \quad -6a = -36 \quad \Rightarrow \quad a = 6.$$

So Elena bought 6 apples and $20 - 6 = 14$ oranges.

Discussion. When solving this kind of problem, we can choose different variables depending on what we find easier — for instance, we could let the variable represent the number of apples or the number of oranges. The equations would look slightly different but lead to the same answer in the end.

Another good strategy is to use the **guess and check** method. Sometimes this method can be **"stupid,"** meaning we just make random guesses and hope to land on the right answer, but it can also be **smart** if we use logic to improve our guesses.

For example, suppose Elena bought all 20 oranges. That would cost $20 \times 15 = 300$ cents, which is 36 cents more than she actually paid. Each time we replace an orange (15 cents) with an apple (9 cents), we

save 6 cents. To save a total of 36 cents, we need to make $36 \div 6 = 6$ substitutions. So, Elena must have bought 6 apples and $20 - 6 = 14$ oranges the same result we found using algebra.

Example 2. A messenger travels 40 km per day. The next day, another messenger starts along the same route, traveling 45 km per day. After how many days will the second messenger catch up?
Solution. After x days, the first messenger has traveled $40x$ km. The second messenger has traveled $45(x-1)$ km (since he started one day later). We want to know when the distances are equal:

$$40x = 45(x-1).$$

Simplify:

$$40x = 45x - 45 \quad \Rightarrow \quad 5x = 45 \quad \Rightarrow \quad x = 9.$$

So the second messenger will catch up after 9 days.

Classwork

1. Solve the equations:
   (a) $3x - 7 = 11$
   (b) $5 - (2x - 3) = 8$
   (c) $4(x-2) + 7 = 3x + 5$
   (d) $2(3x - 4) = 5x - 6$
2. Three consecutive integers have a sum equal to five times the smallest plus 21. Find these integers.

Solutions to Classwork

1. **Solve the equations.**
   (a) $3x - 7 = 11 \Rightarrow 3x = 18 \Rightarrow x = 6.$
   (b) $5 - (2x - 3) = 8 \Rightarrow 5 - 2x + 3 = 8 \Rightarrow 8 - 2x = 8 \Rightarrow x = 0.$
   (c) $4(x-2) + 7 = 3x + 5 \Rightarrow 4x - 8 + 7 = 3x + 5 \Rightarrow 4x - 1 = 3x + 5 \Rightarrow x = 6.$
   (d) $2(3x - 4) = 5x - 6 \Rightarrow 6x - 8 = 5x - 6 \Rightarrow x = 2.$
2. **Three consecutive integers.** Let the integers be n, $n+1$, $n+2$. Then

$$\underbrace{(n + (n+1) + (n+2))}_{=3n+3} = \underbrace{5n}_{\text{five times the smallest}} + 21 \Rightarrow 3n + 3 = 5n + 21 \Rightarrow 2n = -18 \Rightarrow n = -9.$$

   Answer: $-9, -8, -7$

Homework

1. **Simplify each expression.**
   (a) $3(2x-1)$
   (b) $2-(1-x)$
   (c) $7x-(3x+15)$
   (d) $3(2x-1)+x$
   (e) $2a(a-2)-a(a-1)$
   (f) $(2x-1)(x+1)$
2. Alex has \$50. In a store, chocolate bunnies are sold for \$0.45 each. To carry them home, Alex also needs to buy some bags, each costing \$0.30. Each bag can hold up to 30 chocolate bunnies. Alex buys as many bunnies as possible and enough bags to carry them all. How much money does Alex have left?
3. A dog weighs 2 pounds more than a cat. Three cats and four dogs together weigh 43 pounds. How much does a dog weigh? How much does a cat weigh?
4. A father is twice as old as his son. The sum of their ages is 48 years. How old are they?
5. An orange costs 2 cents more than an apple. A grapefruit costs as much as three oranges. A fruit basket consists of 10 apples, 5 oranges, and a grapefruit.
   (a) If the price of an apple is a, what is the price of an orange? of a grapefruit? Simplify your expressions.
   (b) If the fruit basket costs \$1.96, how much does each fruit cost?
6. A pet store sells parrots and canaries. A canary costs twice as much as a parrot. One customer bought 5 canaries and 3 parrots, while another bought 3 canaries and 5 parrots. One of the customers paid \$20 more than the other. How much does each bird cost?
7. In a faraway kingdom, there are two currency exchange offices. The first one gives 300 tugriks per dollar, but charges a 700-tugrik commission for the exchange. The second one gives only 295 tugriks per dollar, but no commission is charged.

   A traveler noticed that it makes no difference to him which office he uses – he would get the same total amount of tugriks either way. How many dollars is he planning to exchange?
8. Two lumberjacks, Jack and Peter, were working together in the forest and sat down to have lunch. Jack had 4 flatbreads, and Peter had 8. Just then, a traveler came by.

   "Hey friends," he said, "I've lost my way in the woods, and the village is still far off. I'm starving could you share a bit of your food with me?"

   "Of course," replied the lumberjacks. "Come join us – we'll share what we have."

   So, the 12 flatbreads were divided equally among the three men. After eating, the traveler checked his pockets, found a \$6, and said:

   "Sorry, friends, that's all I've got. Please share it between yourselves however you think is fair."

   The traveler left, but the lumberjacks started arguing. Peter said, "Let's just split the money evenly." But Jack replied, "Wait — we had 12 flatbreads, that's \$6 total, or about \$0.5 per flatbread. You had 8, I had 4, so I should get one-third, and you two-thirds."

   How would you divide the money between the two lumberjacks?

*9. John and Sally together have 93 cents; John and Mina together have 104 cents; Sally and Mina together have 95 cents. How much money does each of them have?

*10. **Translation problem.** The list below shows some dates written in Swahili:

tarehe tatu Disemba jumamosi
tarehe pili Aprili jumanne
tarehe nne Aprili jumanne
tarehe tano Octoba jumapili
tarehe tano Octoba jumatatu
tarehe tano Octoba jumatano

Here are their English translations (in a different order!):

October 5, Monday
April 2, Tuesday
October 5, Wednesday
October 5, Sunday
December 3, Saturday
April 4, Tuesday

Write the following dates in Swahili: April 3, Wednesday; December 2, Sunday; December 5, Monday.