Math 4e. Classwork 26.

1. Is the number $25^2 \cdot 9^3$ divisible by 5, 3, 15, 20, 45?

2. Show that
   $(57! + 58!)$ is divisible by 59. (Hint: find common factors.)

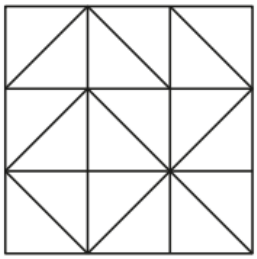
3. Prove that
   $8^5 + 2^{11}$ is divisible by 17
   $9^7 - 3^{10}$ is divisible by 20

4. Compare $25^2 \cdot 9^3$ and $225^2 \cdot 9$

5. Positive or negative value of *m* will make the following equalities true statements?

$\|m\| = m$	$m = -m$
$\|m\| = -m$	$m + \|m\| = 0$
$-m = \|-m\|$	$m + \|m\| = 2m$
$m = \|-m\|$	$m - \|m\| = 2m$

6. A square is divided into triangles (see the figure). How many ways are there to color exactly one third of the square? (The small triangles cannot be colored partially.)



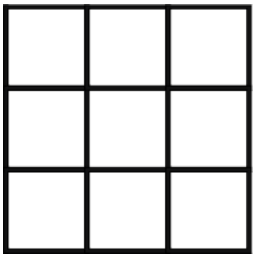
7. Calculate the value of the expression:

   $a.\ \frac{80!}{79!} + \frac{59!}{58!}$; $b.\frac{2025! - 2024!}{2023!}$

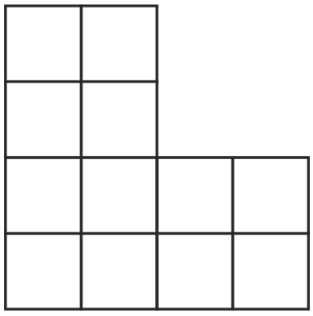
   $c.\ \frac{5 \cdot 4^{27} - 21 \cdot 4^{26}}{2^{50}}$; $d.\ \frac{3^{51} - 4 \cdot 3^{50}}{9^{26}}$;

   $e.\ 101101 \cdot 999 - 101 \cdot 999999$;

8. How can you measure 6 liters of water from the faucet using only 5-liter and 7-liter jars?
9. How many rectangles are there in the picture?



10. Can the figure into 4 identical shapes:



11. In the number 52*2*, replace the asterisks with digits so that the resulting number is divisible by 36. List all possible solutions.

12. There are three identical boxes on a table. One contains 2 black balls, another contains 1 black and 1 white ball, and the third contains two white balls. The boxes are labeled: "2 white," "2 black," and "black and white." It is known that none of the labels correspond to the actual contents. How can you determine the correct arrangement of the labels by drawing only one ball?

13. There are 8 sheep in a flock. The first eats a haystack in 1 day, the second in 2 days, the third in 3 days, ..., the eighth in 8 days. Who will eat a haystack faster: the first two sheep together or all the rest together?

14. Alice and Taylor together weigh 40 kg, Taylor and Maya — 50 kg, Maya and Victor — 90 kg, Victor and Dylan — 100 kg, and Dylan and Alice — 60 kg. How much does Alice weigh?