

1 percent of quantity is a  $\frac{1}{100}$  th part of it.

### Percent and ratio, proportions.

One percent (1%) means 1 per 100. Percentage is a ratio expressed as a number out of 100. For example, if 20% of the students in grade 4 got a perfect score on the test, it means that 20 out of every 100 students got a perfect score.

$$\frac{20}{100} = \frac{1}{5}$$

Or every 5<sup>th</sup> student got a perfect score.

1% of this line is shaded green: it is very small, isn't it?



To cook a raspberry jam according to recipe I need to combine 3 cups of berries and 2 cups of sugar, or for each 3 cups of raspberries go 2 cups of sugar; ratio of raspberries and sugar (in volume) is 3: 2. If I bought 27 cups of raspberries, how many cups of sugar do I need to put to my jam?

$$\frac{3}{2} = \frac{27}{x}$$

Two ratios which are equal form a proportion.

Proportions have several interesting features.

1. The products of inside and outside terms are equal.

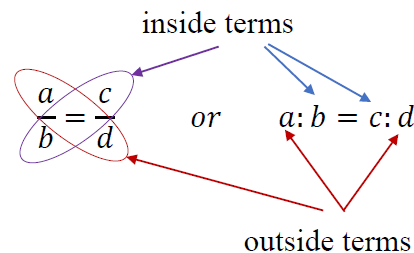
$$\frac{a}{b} = \frac{c}{d} \Leftrightarrow a \cdot d = b \cdot c$$

It can be easily shown:

$$\frac{a}{b} = \frac{c}{d} \Leftrightarrow \frac{adb}{b} = \frac{cdb}{d} \Leftrightarrow ad = cb$$

2. Also, two inverse ratios are equal:

$$\frac{a}{b} = \frac{c}{d} \Leftrightarrow \frac{b}{a} = \frac{d}{c}$$



Indeed:

$$\frac{a}{b} = \frac{c}{d} \Leftrightarrow a \cdot d = b \cdot c \Leftrightarrow \frac{ad}{ac} = \frac{bc}{ac} \Leftrightarrow \frac{d}{c} = \frac{b}{a}$$

Percent can be seen as a ratio to a hundred:

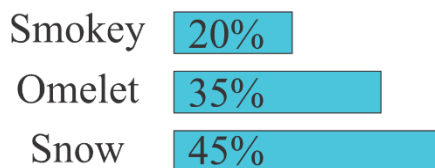
$$\frac{1}{2} = \frac{50}{100} = 0.5 = 5\%$$

### Exercises:

1. During the year, the price of strudels was increased twice by 50%, and before New Year they were sold at half price.

What is the current price of one strudel, if at the beginning of the year it cost 4 dollars?

2. After the crisis, all prices increased by 25%. By what percentage can fewer goods be bought with the same salary?
3. At what interest rate is it more profitable to deposit money in a bank for a year: 6% per year or 0.5% per month?
4. In one of the communities of a social network, there was a vote on which kitten in the photo is the cutest. By the morning, the votes were distributed as follows:



By the evening, more votes were added, but all the new votes were for Snow. As a result, Smokey was left with only 16% of the votes. What percentage of the votes did Omelet have in the evening?

5. Three pirates were dividing a bag of coins. The first took  $\frac{3}{7}$  of all the coins, the second took 51% of the remaining coins, and the third received 8 coins fewer than the second. How many coins were there in the bag?

6. Dry mushrooms contain 10% water. Fresh mushrooms contain 95% water. What was the weight of the fresh mushrooms if 30 g of dry mushrooms were obtained from them?
7. Sea water contains 5% salt (by weight). How many kilograms of fresh water should be added to 40 kg of sea water to obtain a solution with 2% salt?
8. There was milk in one glass and the same amount of coffee in another. One spoonful of milk was poured into the glass with coffee and stirred. Then the same spoonful of the mixture was poured back into the glass with milk. Now, which is greater: the amount of coffee in the milk glass or the amount of milk in the coffee glass?
9. A merchant accidentally mixed candies of the first type (priced at \$3 per pound) with candies of the second type (priced at \$2 per pound). At what price should this mixture be sold to obtain the same total amount, given that it is known that initially the total cost of all candies of the first type was equal to the total cost of all candies of the second type?

10. How many squares we have to shade to shade 10% of the line, 15%, 20%, 25%?



11. There are 200 pencils in the box. 3% of the pencils are red, 26% are yellow, and the rest are blue. How many red pencils are in the box?

12. Find:

1% from 100

120% from 250

7% from 200

5% from 50

100% from 49

25% from 48

1% from 300

200% from 300

20% from 15

13. Find a number, if

1% of it is 2;

10% of it is 12;

15% is 150;

3% of it is 0.24;

200% of it is 400;

100% of it is 0.1;

50% of it is 1;

25% of it is 30;

14. 15% of the participant of math Olympiad solved 1 problem, 25% of the participant solved 2 problems, and the rest 24 students solved all three problems. How many students did participate in the math Olympiad?

15. Dry apricots contain 25% of water. How much water should be evaporated from 5 kg of fresh apricots to get dry apricots, if fresh apricots contain 85% of water?

16. Peter spent 15% of his money and 1.5 dollars on a doughnut and  $\frac{3}{5}$  of his money and 30 cents on ice-cream. How much money did he have?

17. In Peter's bottle there is 10% more soda than in John's bottle. Peter drank 11% of his soda, while John drank 2% of his soda. So, who has more soda left?

18. Write an expression to find 15% of a number  $a$ . Calculate 15% of the following numbers: 1540, 220, and 10.

19. Write an expression to find a number, if 4% of it is equal to  $b$ . Find the numbers for which 4% is equal to 8, 12, and 55.