Math 4e. Classwork 21.

If a number a in a power n is divided by the same number in a power m,

$$\frac{a^n}{a^m} = \frac{\underbrace{a \cdot a \cdot \ldots \cdot a}_{n\ times}}{\underbrace{a \cdot a \cdot \ldots \cdot a}_{m\ times}} = \left(\underbrace{a \cdot a \cdot \ldots \cdot a}_{n\ times}\right) : \left(\underbrace{a \cdot a \cdot \ldots \cdot a}_{m\ times}\right) = a^n : a^m = a^{n-m}$$

So, we can say that

$$a^{-1} = \frac{1}{a^1} = \frac{1}{a}; \qquad a^{-n} = \frac{1}{a^n};$$

Let's see how our decimal system of writing numbers works when we use the concept of exponent: $3456 = 1000 \cdot 3 + 100 \cdot 4 + 10 \cdot 5 + 1 \cdot 6 = 103 \cdot 3 + 102 \cdot 4 + 101 \cdot 5 + 100 \cdot 6$

The value of a place of a digit is defined by a power of 10 multiplied by the digit. Very large numbers can be written using this system, as well as very small numbers.

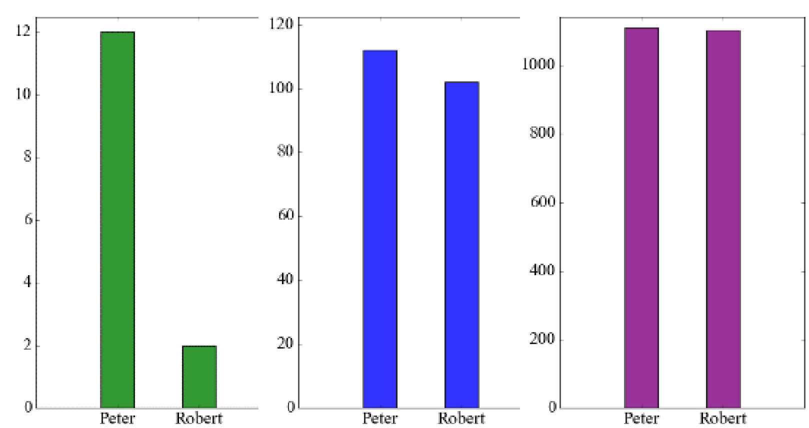
$$0.3 = \frac{1}{10} \cdot 3 = 10^{-1} \cdot 3;$$

$$0.456 = \frac{1}{10} \cdot 4 + \frac{1}{100} \cdot 5 + \frac{1}{1000} \cdot 6 = 10^{-1} \cdot 4 + 10^{-2} \cdot 5 + 10^{-3} \cdot 6$$

Ratio.

Peter has 10 dollars more than Robert. Is this a big difference? How we can compare the amount of money they have?

Take a look at the table



Peter	\$12	\$112	\$1112
Robert	\$2	\$102	\$1102

In all these cases the absolute difference is the same, but in the first case Peter has 6 times as much as Robert, in the last situation they both have almost the same amount of money. The ratios of the amount of Peter's money and Robert's money are.

$$\frac{12}{2}; \qquad \frac{112}{102}; \qquad \frac{1112}{1102};$$

The amount of money Peter and Robert have in the first case is 12 and 2 dollars and the ratio is $\frac{12}{2} = 6$, or 6:1, or 6 to 1.

The ratio of two numbers indicates how many times one number is larger than another or which part of one number the other number is.

Example1: (it's not a real recipe) The ratio of water and lemon juice in lemonade is 4 to 1. What does it mean? In means that for each part of lemon juice we need to add 4 parts of water. For example, for 1 liter of lemon juice, 4 liters of water are needed. If there is only 1 cup of lemon juice, 4 cup of water should be added. Therefore, the total volume should contain 5 equal parts, $\frac{1}{5}$ of the total volume is juice and $\frac{4}{5}$ is water.

How much of lemon juice and water we need to prepare 1 l. of lemonade?

If we want to have sweet lemonade and we add sugar. The ratio of water, lemon juice and sugar is 4:1:0.5 (or it can be rephrased as 8:2:1). For each part of sugar, we will use 2 parts of lemon juice, and 8 parts of water.

We can write the ratio of two numbers in the several ways:

$$a \text{ to } b, \qquad a{:}\,b, \qquad \frac{a}{b}$$

Example2: To make pancakes we use 3 cups of flour and 2 cups of milk.

So, the ratio of flour to milk is 3 : 2**,** which means that for each 2 cups of milk we need to have 3 cups of flour. To make pancakes for a LOT of people we might need 4 times the quantity, so we multiply the numbers by 4:

$$(3\cdot 4):(2\cdot 4) \;=\; 12:\;8 \qquad \left(\frac{3\cdot 4}{2\cdot 4}=\frac{12}{8}\right)$$

In other words, 12 cups of flour and 8 cups of milk.

The ratio is still the same, so the pancakes should be just as yummy.

Example3:

Three brothers, 5, 7, and 9 years old went to trick-o-treat. They got 84 sweets altogether. They decided to divide the candies in the ratio of their age 9:7:5. How many candies each of them should get?

To divide all candies between the brothers we need to find the "unit" part of the total amount of candies. The oldest brother should get 9 of such units, the middle one should get 7, and the youngest brother will get 5. Total amount of units is $9 + 7 + 5 = 21$. The number of candies is 84, so the "unit" contains $84: 21 = 4$ candies.
So, the first brother will get $4 \cdot 9 = 36$ candies, the second will get $4 \cdot 7 = 28,$ and the third will get $4 \cdot 5 = 20.$ $36 + 28 + 20 = 88$

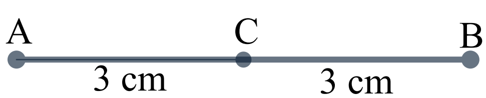
Example 4.

Draw the segment $[AB] = 6cm$. Mark the point C in such a way that the ratio of the length of the segments are

$a.\ \frac{|AC|}{|BC|} = 1,\qquad b.\ \frac{|AC|}{|BC|} < 1;\qquad c.\ \frac{|AC|}{|BC|} > 1;\qquad d.\ \frac{|AC|}{|BC|} = 2$

First, we need to draw a segment 6 cm long. Use ruler!

a. The lengths of the segments [AC] and [CB] should be the same, because the ratio is 1. So, we have to mark our point C right in the middle.

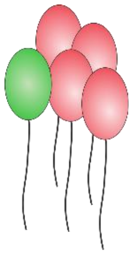


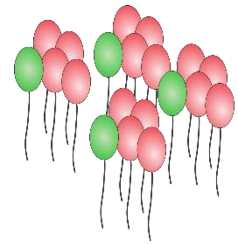
b. Length of the segment [AC] should be smaller than the length of the segment [CB].

c. Length of the segment [AC] should be greater than the length of the segment [CB]

d. The segment [AC] should be twice as long as the segment [CB]: |AC|=4 cm, |CB|=2 cm.

Example 5: There are 1165 red and green balloon in the store. The ratio of GB to RD is 1 to 4. How many green and red balloon are in the store? We can have two point of view on this problem:





Ratio of the green balloon to the red balloons is $\frac{1}{4}$ (1 to 4). On both pictures there are four red balloons for each green balloon. The number of green balloons is a quarter of the number of red balloons. (Or, the number of RB is 4 times greater than the number of GB).

$$\frac{1}{4} = \frac{4}{16}$$

1. Because there are 4 RB for each green one, all balloons can be divided into groups of 5 : 1 green and 4 red. $1165 : 5 = 233$. So, there are 233 green balloons and $4 \cdot 233 = 932$ red balloons.

2. $\frac{1}{5}$ part of all balloons will be green balloons, $4 + 1 = 5$, and $\frac{4}{5}$ will be red. $\frac{1}{5} \cdot 1165 == 233$

Also, we can say that the ratio of green balloons to all balloons is $\frac{1}{5}$, or 1 to 5, in other words, one out of 5 balloons is green (and therefore 4 are red)
The ratio of quantities of the same kind is a number: ratio of the number green balloons to the red ones is a number, this number shows how many times red balloons more than green.
The ratio of quantities of different kinds forms a new quantity. For example, speed v is the ratio of distance traveled S to travel time t. S is measured in length units, time is measured in the time units, therefore speed is measured in the distance unit per time unit. If I need 5 hours to cover the distance of 500 km, my speed is going to be

$$v = \frac{500\ km}{5\ h} = 100\ \frac{km}{h} = 100\ kilometers\ per\ hour$$

Exercises:

1. Simplify the expressions:

$a.\ \ 2^4 + 2^4;$ $\qquad b.\ \ 2^m + 2^m;$ $\qquad c.\ \ 2^m \cdot 2^m;$

$d.\ \ 3^2 + 3^2 + 3^2;$ $\qquad e.\ \ 3^k + 3^k + 3^k;$ $\qquad f.\ \ 3^k \cdot 3^k \cdot 3^k;$

2. What will be last digit of

$a.\ \ 2^{22};$ $\qquad b.\ 3^{33};$ $\qquad c.\ \ 4^{44};$ $\qquad d.\ 5^{55};$ $\qquad e.\ 6^{66};$ $\qquad f.\ 7^{77};$

3. Compare:
   Example: What is greater 31^{11} or 17^{14}?
   We can see that $31 < 32 = 2^5$; $2^4 = 16 < 17$,

$$31^{11} < 32^{11} = (2^5)^{11} = 2^{55}$$

$$(17)^{14} > 16^{14} = (2^4)^{14} = 2^{56}$$

   We can write the following:

$$31^{11} < 32^{11} = 2^{55} < 2^{56} = 16^{14} < (17)^{14}$$

$$31^{11} < 17^{14}$$

   a. 127^{23} and 513^{18}

   b. 9997^{10} and 100003^{8}

   c. 5^{300} and 3^{500}

4. Write the following numbers as the power of base 10:

   10, 100, 1000, 10000, 100000, 1000000

   0.1, 0.01, 0.001, 0.0001, 0.00001, 0.000001

5. In a box, there are 25 red, 30 yellow, and 15 blue balls. Find the ratio of:
   a. the number of red balls to the number of yellow balls;
   b. the number of yellow balls to the number of blue balls;
   c. the number of red balls to the number of blue balls;
   d. the number of red balls to the total number of all balls.
   e. the number of yellow balls to the total number of all balls.
   f. the number of blue balls to the total number of all balls.

6. **A lot or a little:**
   a) 5 math lessons in one day and in one month;
   b) a weight increase of 1 gram for an ant and for an elephant.

   Come up with your own examples when the same value gives a different qualitative evaluation of a certain situation.

7. $a.$ Draw two line segments whose lengths are in the ratio 2:3.
   $b.$ Draw a rectangle whose side lengths are in the ratio 5:3.
   $c.$ Draw two different rectangles, so that the ratio of the bigger side to the smaller side is 3 to 2.

8. In a dried fruit mix, there are 7parts of dried apples, 4 parts of dried pears and 5 parts of dried apricots. (So, it can be said, that the quantity of apples, pears, and apricots should be mixed in the ratio 7:4:5). What is the weight (how many grams) of apples, pears, and apricots in the fruit mix, if the total weight of the mix is 1600g?

9. In order to prepare a homemade dried fruits and nuts mix Mary took 6 parts of raisins, 5 parts of dried cranberries and 3 parts of walnuts. Cranberries and walnuts altogether weighted 2 kg 400 g. What was the weight of the mix that Mary prepared?

10. The ratio of cashews and walnuts in a nut mixture is 2:3, total weight of the mixture 150g. How much cashews and walnuts are in the pack of mixture?

11. The ratio of roses and hibiscuses in the garden is 9:11. What is the total number of flower bushes in the garden, if there are 99 rose bushes?