Math 4e. Class work 20.

Exponent.

Exponentiation is a mathematical operation, written as a^n, involving two numbers, the base *a* and the exponent *n*. When *n* is a positive integer, exponentiation corresponds to repeated multiplication of the base: that is, a^n is the product of multiplying *n* bases:

$$a^n = \underbrace{a \cdot a \cdot a \ldots \cdot a}_{n\ times}$$

In that case, a^n is called the *n*-th power of *a*, or *a* raised to the power *n*.
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The exponent indicates how many copies of the base are multiplied together. For example, $3^5 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 243$. The base 3 appears 5 times in the repeated multiplication, because the exponent is 5. Here, 3 is the *base*, 5 is the *exponent*, and 243 is the *power* or, more specifically, *the fifth power of 3*, *3 raised to the fifth power*, or *3 to the power of 5*.

Properties of exponent:

$$a^{10} \cdot a^{15} = \underbrace{a \cdot a \ldots \cdot a}_{10\ times} \cdot \underbrace{a \cdot a \ldots \cdot a}_{15\ times} = \underbrace{a \cdot a \cdot a \ldots \cdot a}_{10+15\ times} = a^{10+15} = a^{35}$$

$$(a^{10})^{15} = \underbrace{a^{10} \cdot a^{10} \cdot \ldots \cdot a^{10}}_{15\ times} = \underbrace{\underbrace{a \cdot a \cdot \ldots \cdot a}_{10\ times} \cdot \ldots \cdot \underbrace{a \cdot a \cdot \ldots \cdot a}_{10\ times}}_{15\ times} = a^{10 \cdot 15} = a^{150}$$

$$a^{10} \cdot a = \underbrace{a \cdot a \cdot a \ldots \cdot a}_{10\ times} \cdot a = \underbrace{a \cdot a \cdot a \cdot a \ldots \cdot a}_{10+1\ times} = a^{10+1} = a^{10} \cdot a^1$$

In order to have the set of power properties consistent, $a^1 = a$ for any number *a.*

$$a^{10} = a^{10} \cdot 1 = a^{10+0} = a^{10} \cdot a^0$$

In order to have the set of properties of exponent consistent, $a^0 = 1$ for any number *a,* but 0.
Also, if there are two numbers *a* and *b*:

$$(a \cdot b)^{10} = \underbrace{(a \cdot b) \cdot \ldots \cdot (a \cdot b)}_{10\ times} = \underbrace{a \cdot \ldots \cdot a}_{10\ times} \cdot \underbrace{b \cdot \ldots \cdot b}_{10\ times} = a^{10} \cdot b^{10}$$

1. $a^n = \underbrace{a \cdot a \cdot a \dots \cdot a}_{n\ times}$
2. $a^n \cdot a^m = a^{n+m}$
3. $(a^n)^m = a^{n \cdot m}$
4. $a^1 = a$, for any a
5. $a^0 = 1$, for any $a \neq 0$
6. $(a \cdot b)^n = a^n \cdot b^n$

- A positive number raised into any power will result a positive number.
- A negative number, raised in a power, represented by an even number is positive, represented by an odd number is negative.

Exponents can help write numbers in their expanded form. In our base-10 place value system, the position of the digit indicates which power of 10 should be multiplied by the digit. For example:

$$345 = 300 + 40 + 5 = 100 \cdot 3 + 10 \cdot 4 + 1 \cdot 5 = 10^2 \cdot 3 + 10^1 \cdot 4 + 10^0 \cdot 5$$

Exponent is very interesting mathematical operation. There is the story of the invention of the game of chess. The king ordered a new game because he was bored by the old games, was so happy about the new chess game that he said to the inventor: "*Name your reward and you will get it!*" The inventor asked for a simple reward. *"I would like to have one grain of rice on the first chess square, two on the second, four on the third and so on, doubling the amount of rice every square.*" The legend says that the King was surprised he didn't ask for gold but was quite content that the inventor asked for so little. But when the court scholars told him there wasn't enough rice in the whole world to fill the chess board, he had to admit his loss:

$1 + 2 + 2^2 + 2^4 + \cdots + 2^{63} = 18{,}446{,}744{,}073{,}709{,}551{,}615$

The weight of the rice grain is about 0.03g. so:

$$1.8 \cdot 10^{19} \cdot 0.03 = 5.4 \cdot 10^{17}\ g.\ or\ about\ 5.4 \cdot 10^{14} kg\ or\ 10^{15} lb.$$

Exercises:

1. Continue the sequence:

   $a.$ $1, 4, 9, 16 \dots$ $b.$ $1, 8, 27, \dots$ $c.$ $1, 4, 8, 16 \dots$ $d.$ $1, 3, 9, 27 \dots$

2. What should be the exponent for the equation to hold?

   $a.$ $8^* = 512;$ $b.$ $2^* = 64;$ $c.$ $3^* = 81;$ $d.$ $7^* = 343$

3. What digits should be put instead of * to get true equality? How many solutions does each problem have?

   $a.$ $(2*)^2 = ** 1;$ $b.$ $(3*)^2 = *** 6$

   $c.$ $(7*)^2 = *** 5$ $d.$ $(2*)^2 = ** 9,$ $e.$ $(3*)^2 = ** 1$

4. In a magical lake, the number of water lilies doubles every night. On March 1st, the magician planted the first lily, and in 90 nights, the entire lake was covered with lilies. On which day was only half of the lake covered?

5. Evaluate:

$$(-3)^2; \quad -3^2; \quad (-3)^3; \quad 2^7; \quad (-2)^7; \quad -2^7; \quad (2 \cdot 3)^3; \quad 2 \cdot 3^3; \quad \left(\frac{1}{3}\right)^2; \quad \frac{1}{3^2};$$

6. Represent numbers as a power of 10:

   Example: $1000^3 = (10^3)^3 = 10^{3 \cdot 3} = 10^9$

   $100^2; \quad 100^3; \quad 100^4; \quad 100^5; \quad 100^6;$

7. Write the number which extended form is written below;

   Example: $2 \cdot 10^3 + 7 \cdot 10^2 + 2 \cdot 10^1 + 6 \cdot 10^0 = 2726;$

   $a.\ 2 \cdot 10^3 + 4 \cdot 10^2 + 5 \cdot 10^1 + 8 \cdot 10^0;$

   $b.\ 7 \cdot 10^3 + 2 \cdot 10^2 + 0 \cdot 10^1 + 1 \cdot 10^0;$

   $c.\ 9 \cdot 10^3 + 3 \cdot 10^1 + 3 \cdot 10^0;$

   $e.\ 4 \cdot 10^3 + 1 \cdot 10^2 + 1 \cdot 10^1 + 4 \cdot 10^0;$