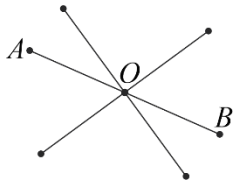
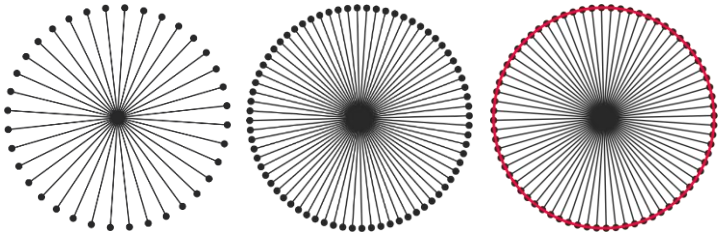
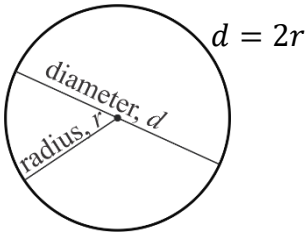
Math 4e. Class work 19.

Circles.

Take a look on the picture:
Point O is a midpoint (center) of a segment [AB], as well as all other segments. All segments are equal, so the distance from the points (A, B, and other) to point O is the same. Let's draw more such segments, equal to [AB] and having point O as their center:







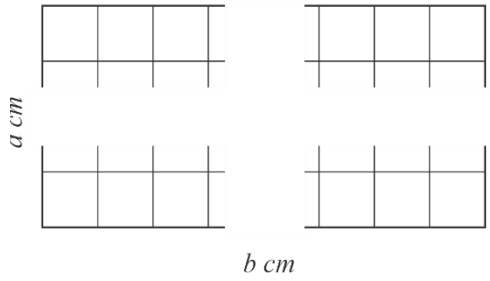
If we continue to draw segments infinitely, we will mark all the points equidistant from O, such shape we call a circle.

Distance from center to the circle is called radius and the segments, connecting two points on a circle and passing through the center is called diameter, the length of the diameter is equal to two radii.

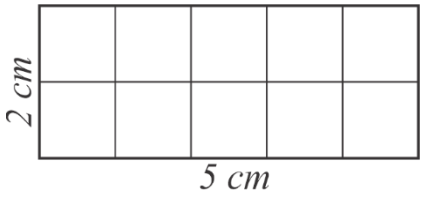
Perimeter and area of some polygons can be calculated relatively easy, area of a rectangle is the product of its two adjacent sides, and perimeter is a twice the sum of these sides.

$$P = 2 \cdot (2 + 5); \qquad S = 2 \cdot 5$$

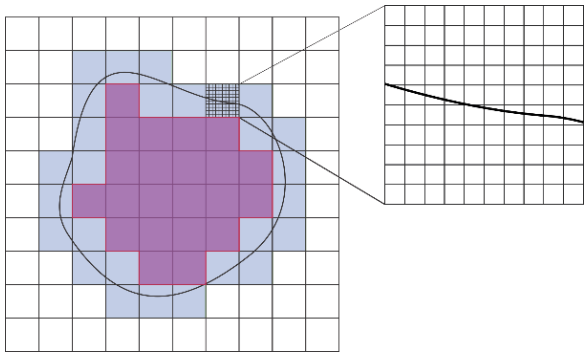
Or, for general rectangle:



$P = 2 \cdot (a + b); \; S = a \cdot b$



Find the perimeter and area of the shape, limited by the closed curve is much more difficult problem. Without using calculus, we can only do it with certain precision. The area of the shape on the picture is
$22 < S < 49$ square units.
Each square the curved line divided into parts can be divided into 100 (10×10) smaller squares and we can find the area more accurate, but still not exactly. This process can be continued forever.



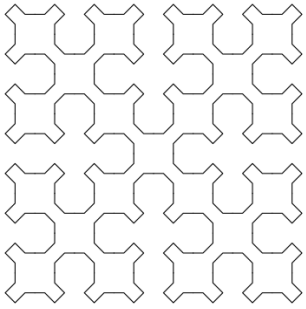
Let's do the experiment: take a paper strip, put it around a mug, a can of beans, or something similar. Cut the paper so that the length of the paper strip is equal to the circumference. Measure the diameter with the ruler, then divide the circumference by the diameter. What did you get? Try to do the same trick with a few more other similar objects. Write down the result of the division of the circumference by a diameter. If you did your measurements accurately, you should get a number close to 3.1 (more exactly, 3.14). Congratulations, you found out, that such ratio is always the same and equal to π ($\approx$ 3.14). π is an infinite, non-periodical decimal, the ratio of a circumference to a diameter of any circle. Using π we can find the circumference and the area of a circle:

$$L = 2\pi r = \pi d; \quad S = \pi r^2$$

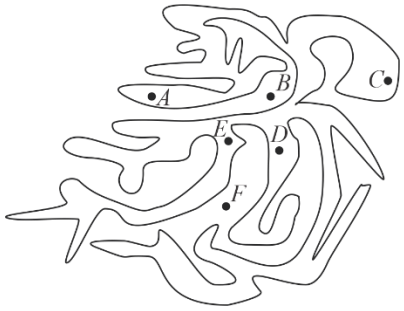
were r being a radius of a circle, d is a diameter.

Exercises:

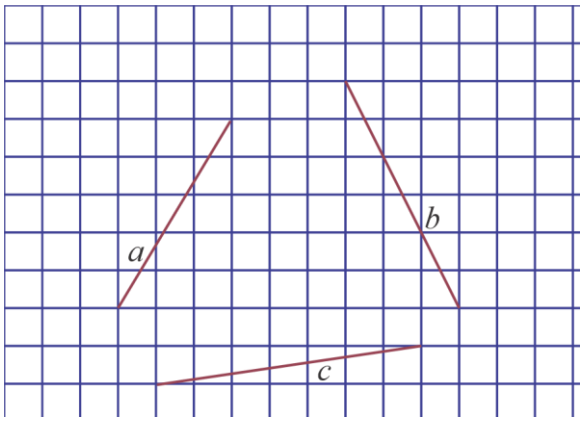
1. Is the curve on the picture closed or open?



2. Inside or outside of the closed curve points A, B, C, D, E, and F are?

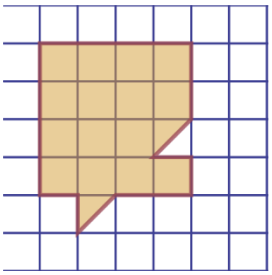


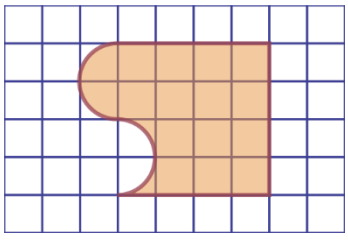
3. Copy the lines a, b, c to your notebook. Using ruler, construct the intersection of these lines. Through the point of intersection of the lines a and c draw line d. (Copy means to draw as shown).



4. In your notebook, mark three points, A, B, C. Draw lines (AB), (AC), (BC). Draw another line, crossing these three lines, use ruler.

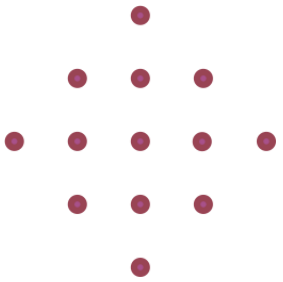
5. Find the area of a shaded shapes (area of a grid unit is 0.25 square centimeter). All lines are drawn along the grid or between the grid nodes. Curved lines are half circles.

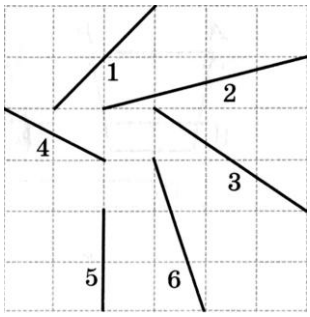




6. Points A, B, and C lie on the same straight line. The distance between points A and B is 20 cm, and the distance between points B and C is 5 cm. Find the distance between points A and C.

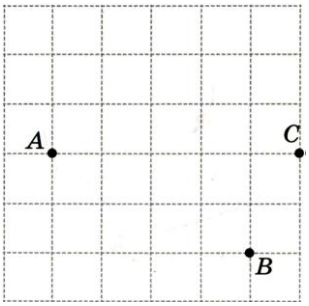
7. On a figure, there are 13 points. How many squares with vertices at these points can be drawn? (All points are located in the nodes of the grid)

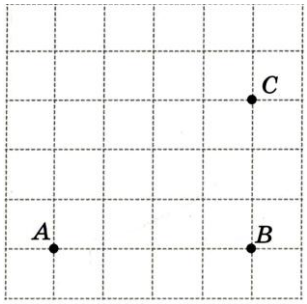




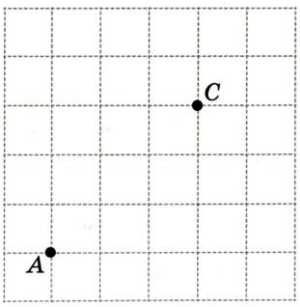
8. Arrange the numbers in increasing order of the lengths of the corresponding segments, without measuring them.

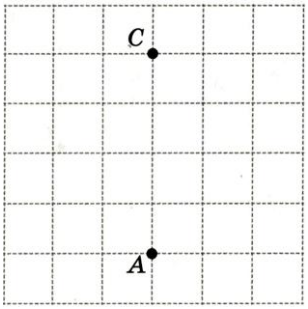
9. Draw a rectangle with the three vertices A, B, and C.



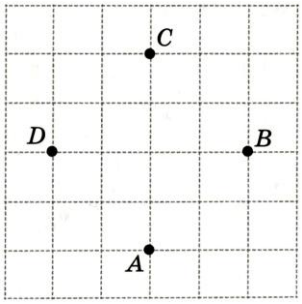


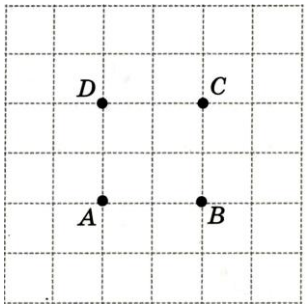
10. Draw a square with the opposite vertices A and C.



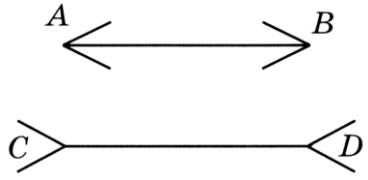


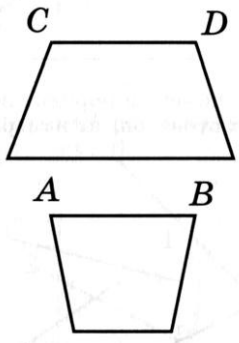
11. Draw a square where points A, B, C, D are midpoints of the side of the square.

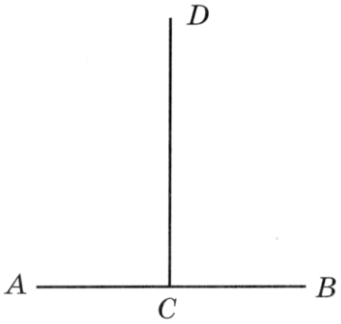




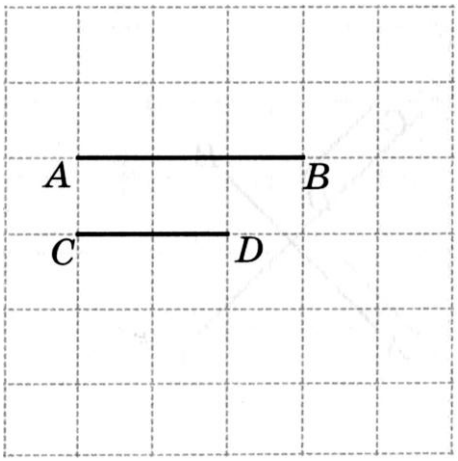
12. Compare the lengths of segments AB and CD without measuring them.

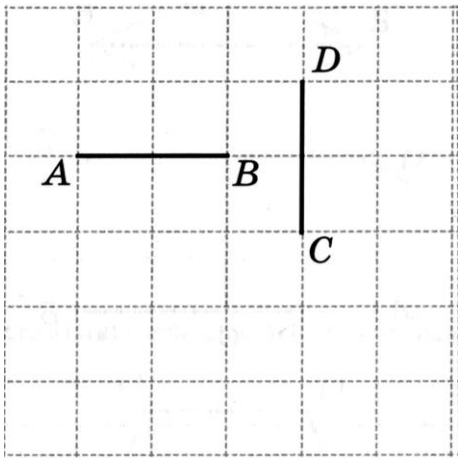


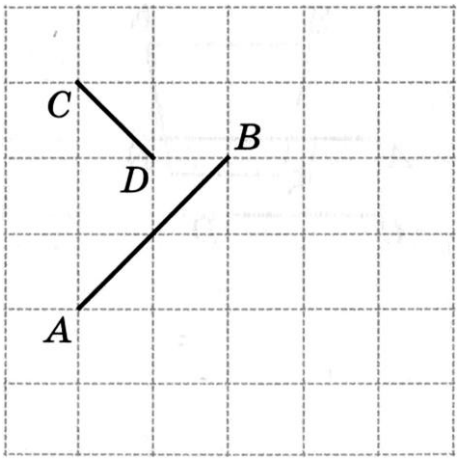




13. Draw a segment equal to the sum of segments AB and CD.







14. Draw a segment equal to the difference of segments AB and CD.