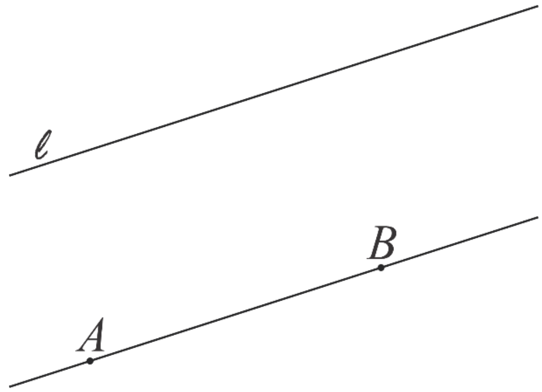
Math 4e. Class work 18.

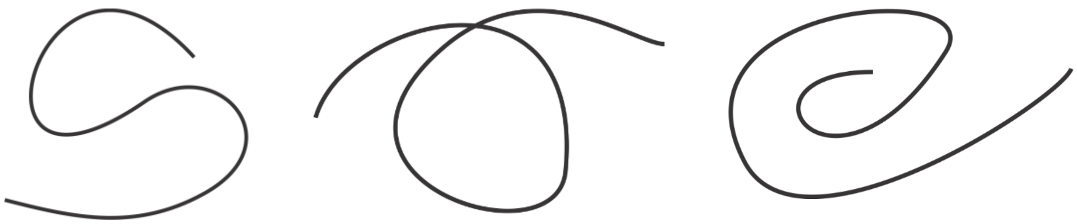
Drawing on a graph paper.

Take a ruler and draw a (straight) line. Try to be accurate. The trace of your pencil on paper represents a (straight) line, a geometrical object. We usually mark lines with the small letter, line l (or with two capital letters):



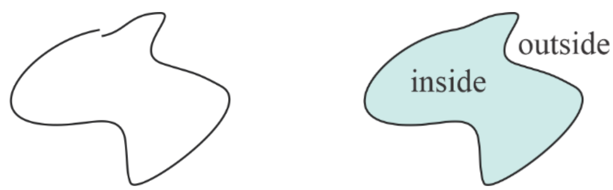
The word “straight” can be omitted, “line” means “straight” line.

f you are freely moving you pencil on paper, you are going to get a curved (or bent) line. It has a curvature, bindings are random. Examples of the curved lines:



If a line and a curve line are represented by roads, on a straight road car is always moves in the same (or opposite) direction, but on the curvy road, a car needs constantly change the direction.

Curved lines can be open and closed. On the picture, which one is open and closed curves?

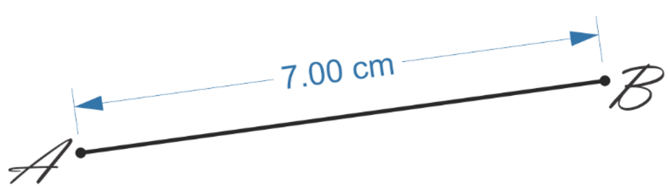


Curved lines can be simple and non-simple; non-simple curved lines have self-crosses. They also can be open and closed.

Mark two points on a paper. Try to connect them by a few different ways. Which one will be the shortest one?

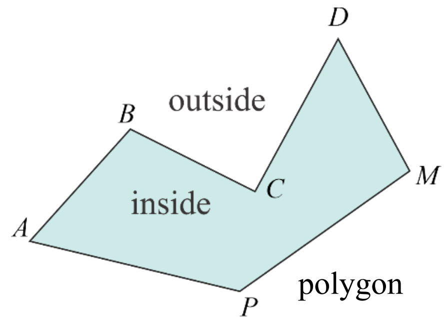
Mark two points on the paper, connect them using your ruler. You have got a segment. Mark the points with capital letters, for example, A and B.

Segment [AB] is drawn on your paper. The distance between the ends of the segment can be measured with ruler, we can find how many units of length (in our case, centimeters) can be fit between the ends.



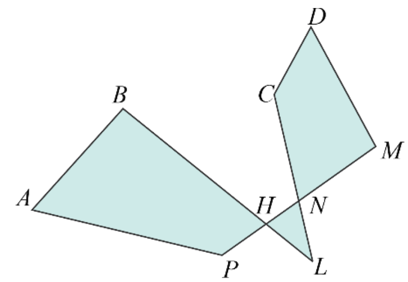
Polygons and circles.

The series of segments can be drowned in a way that the starting point of each segment is an ending point of a previous one. Also, they can be assembled so that the end of the last segment will be the starting point of the first one, like this:



We created a closed area, limited by segments, a polygon. Note, to be a polygon, closed series of segments shouldn't have self-crossings. The ends of the segments are vertices of the polygon and the segments themselves are sides of the polygon. The polygon on the picture has vertices A, B, C, D, M, and P and sides [AB], [BC], [CD], [DM], [MP], [PA].

In geometry, we will study the properties of a various polygons, such as triangles and quadrilaterals. Each polygon has a perimeter, the sum of the length of all sides and an area – measure how many units of the area (square centimeter for example) are covered by the shape.



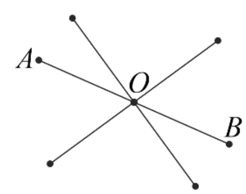
Not a polygon

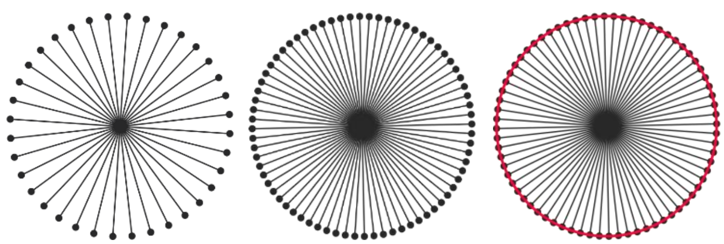
Perimeter is the sum of the lengths of all sides of a polygon.

Area is the measure of how much surface a two-dimensional shape covers.

Take a look on the picture:

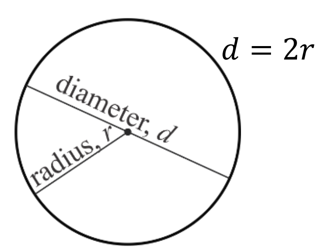
Point O is a midpoint (center) of a segment [AB], as well as all other segments. All segments are equal, so the distance from the points (A, B, and other) to point O is the same. Let's draw more such segments, equal to [AB] and having point O as their center:





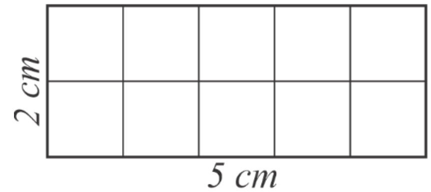
If we continue to draw segments infinitely, we will mark all the points equidistant from O, such shape we call a circle.

Distance from center to the circle is called radius and the segments, connecting two points on a circle and passing through the center is called diameter, the length of the diameter is equal to two radii.

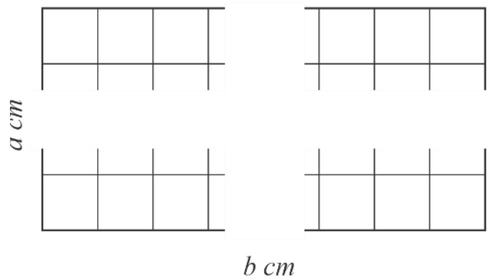


Perimeter and area of some polygons can be calculated relatively easy, area of a rectangle is the product of its two adjacent sides, and perimeter is a twice the sum of these sides.

$P = 2 \cdot (2 + 5); \quad S = 2 \cdot 5$

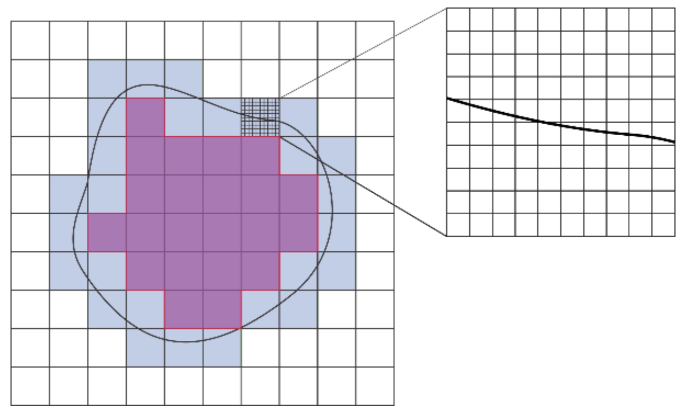


Or, for general rectangle:



$P = 2 \cdot (a + b); \ S = a \cdot b$

Find the perimeter and area of the shape, limited by the closed curve is much more difficult problem. Without using calculus, we can only do it with certain precision. The area of the shape on the picture is
$22 < S < 49$ square units.
Each square the curved line divided into parts can be divided into 100 (10×10) smaller squares and we can find the area more accurate, but still not exactly. This process can be continued forever.



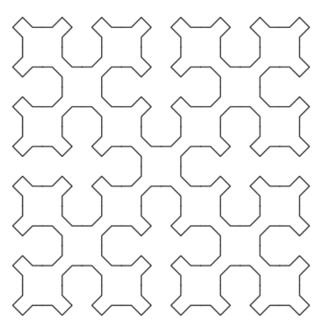
Let's do the experiment: take a paper strip, put it around a mug, a can of beans, or something similar. Cut the paper so that the length of the paper strip is equal to the circumference. Measure the diameter with the ruler, then divide the circumference by the diameter. What did you get? Try to do the same trick with a few more other similar objects. Write down the result of the division of the circumference by a diameter. If you did your measurements accurately, you should get a number close to 3.1 (more exactly, 3.14). Congratulations, you found out, that such ratio is always the same and equal to π (≈ 3.14). π is an infinite, non-periodical decimal, the ratio of a circumference to a diameter of any circle. Using π we can find the circumference and the area of a circle:

$$L = 2\pi r = \pi d; \quad S = \pi r^2$$

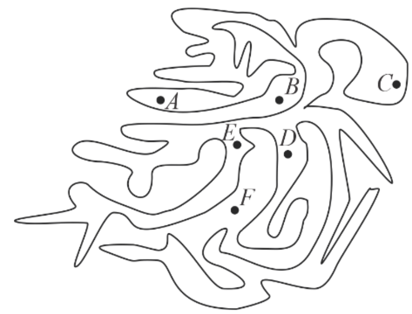
were *r* being a radius of a circle, *d* is a diameter.

Exercises:

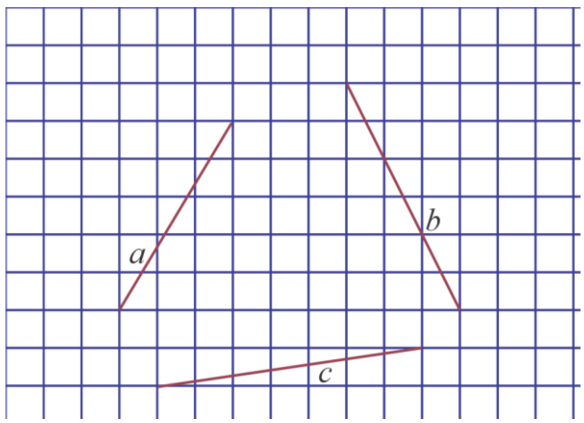
1. Is the curve on the picture closed or open?



2. Inside or outside of the closed curve points A, B, C, D, E, and F are?

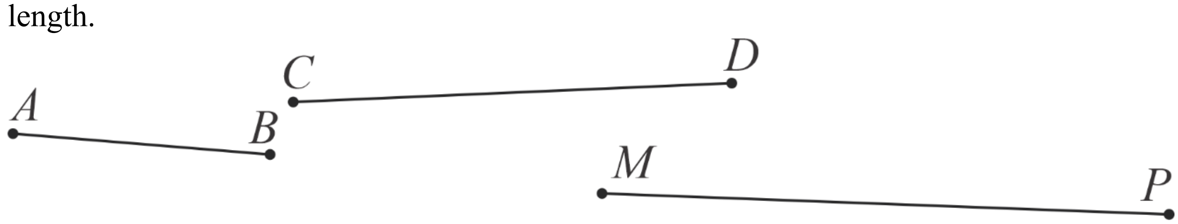


3. Copy the lines a, b, c to your notebook. Using ruler, construct the intersection of these lines. Through the point of intersection of the lines a and c draw line d. (Copy means to draw as shown).



4. In your notebook, mark three points, A, B, C. Draw lines (AB), (AC), (BC). Draw another line, crossing these three lines, use ruler.

5. Take a look on a ruler. Try to guess the length of the segments on the picture. Write the guessed lengths of all three segments. Measure them with a ruler (in centimeters). Write down the measured lengths too. Find the difference between the measured length and guessed length.

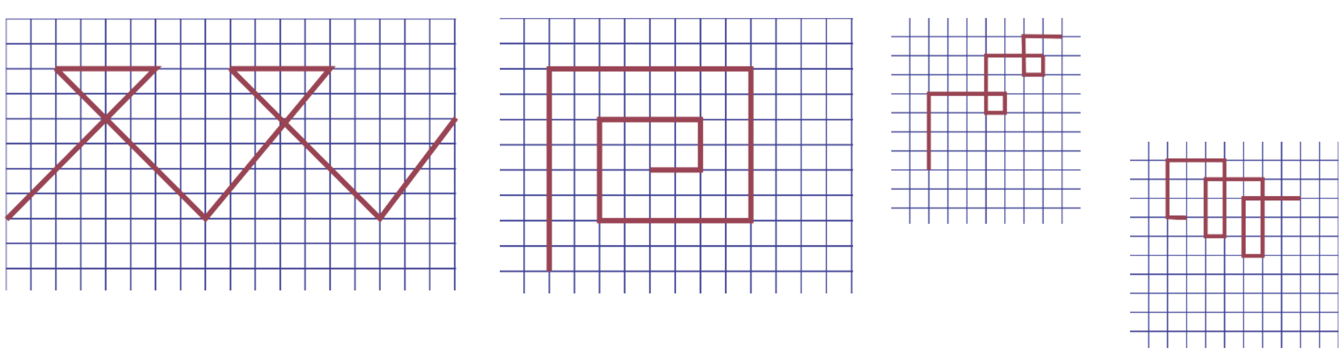


6. Mark point **O** at a grid intersection in a square-ruled notebook. Construct:
   a. Point **A**, located **5 squares to the right** and **4 squares above** point **O**;
   b. Point **B**, located **3 squares to the right** and **2 squares below** point **O**;
   c. Point **C**, located **4 squares to the left** and **1 square below** point **O**.
   d. Connect each of the points **A, B, C** to point **O**.

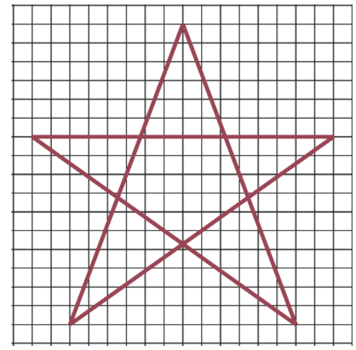
   Name the resulting line segments.

7. Draw the segment AB. Mark a point K so that points A, B, and K do not lie on the same straight line. Through point K, draw:
   a) a line b that intersects the segment AB;
   b) a line d that does not intersect the segment AB.

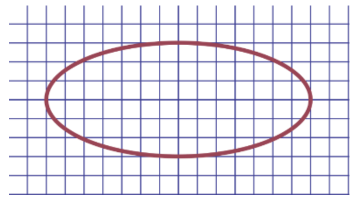
8. In your notebook copy the picture and continue, use ruler:



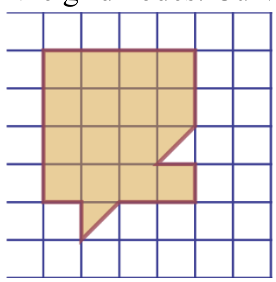
9. Copy the pictures into your notebook:

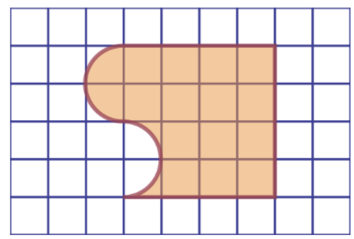


10. Try to copy the ellipse into your notebook.



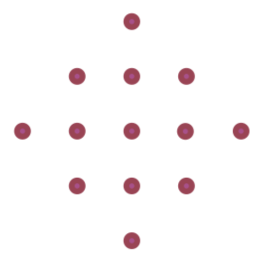
11. Find the area of a shaded shapes (area of a grid unit is 0.25 square centimeter). All lines are drawn along the grid or between the grid nodes. Curved lines are half circles.



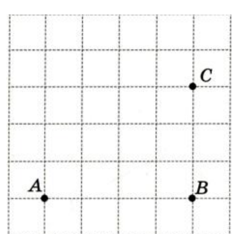


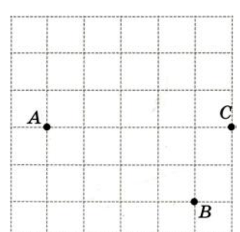
12. Points A, B, and C lie on the same straight line. The distance between points A and B is 20 cm, and the distance between points B and C is 5 cm. Find the distance between points A and C.

13. On a figure, there are 13 points. How many squares with vertices at these points can be drawn? (All points are located in the nodes of the grid)



14. Draw a rectangle with the three vertices A, B, and C.





15. Draw a square with the opposite vertices A and C.

16. Draw a square where points A, B, C, D are midpoints of the side of the square.