Math 4e. Class work 17.

Problem *4. The speed of the boat in a still water on a lake is 12 km/h. The speed of the river flow is 3 km/h. How many hours does the boat need to go from the city A to the city B if the distance between the two cities is 45 km and the city A is up on the river, i.e. the river flows from A to B?*

How many hours does this boat need to go back from the city B to the city A?

If the boat moves along with the river flow, their speeds are combined; in a unit of time the flow will move the boat by 3 km and boat motor will propel it by 12 km, so the speed of the boat downstream is $15\frac{km}{h}$. If the boat goes upstream, flow tries to move the boat down the stream by 3 km. every hour, but the motor propel it by 12 km. in the opposite direction. Total displacement will be $12 - 3 = 9\ km$ in 1 hour, therefore the speed of the boat moving upstream is $9\ \frac{km}{h}$. Time to go from A to B

$$45{:}\,15 = 3h.$$

Time needed to go back from B to A is $45{:}\,9 = 5h$.

Work problems (combine labor problems).

Mary can eat her birthday cake in 10 minutes, and Peter can eat the same cake in 15 minutes. How fast will they eat the same cake together?

These kinds of problems are related to the amount of work done per unit of time; we can call it "rate". To solve the problem, we have to find out what part of the cake Mary will eat in 1 minute. If she can eat the whole cake in 10 minutes, she only eats $\frac{1}{10}$ of the cake in one minute. Peter will eat $\frac{1}{15}$ of the cake in 1 minute. If they will start eating the cake simultaneously, each minute

$$\frac{1}{10} + \frac{1}{15} = \frac{3}{30} + \frac{2}{30} = \frac{5}{30} = \frac{1}{6}$$

Part of the cake will be eaten. We don't know, how many minutes are needed, but the rate with which the cake will be disappearing is $\frac{1}{6}$ per minute:

$$x(minutes) \cdot \frac{1}{6}(part\ of\ the\ cake) = 1(whole\ cake)$$

So, they will need exactly

$$x = 1(whole\ cake){:}\frac{1}{6}(parts) = 1 \cdot 6 = 6\ minutes$$

The bathtub fills with cold water in 6 minutes 40 seconds, with hot water in 8 minutes. Additionally, if the plug is removed from the full bathtub, the water will drain out in 13 minutes and 20 seconds. How much time will it take to fill the bathtub completely, assuming both taps are open but the bathtub is not plugged?

This problem is very similar to a previous one. If only cold-water tap is open, it will take 6 min. and 40 second $\left(6\frac{2}{3} = \frac{20}{3}\right)$ to fill the tub. In one minute $\frac{3}{20}$ of the tub will be filled. If only hot water tap is open $\frac{1}{8}$ of the tab will be filled in one minute. Also, $\frac{3}{40}$ of the tab will be drained out each minute, if the plug is not installed.

$$\frac{3}{20} + \frac{1}{8} - \frac{3}{40} = \frac{6}{40} + \frac{5}{40} - \frac{3}{40} = \frac{8}{40} = \frac{1}{5}$$

$$t \cdot \frac{1}{5} = 1; t = 5 \text{ minutes.}$$

John and Zoe start walking toward each other on a straight road. John is walking at a speed of 3.6 km/h and Zoe is walking at a speed of 2.4 km/h. The distance between the starting points is 12 km. After how many hours will they meet?

In a previous chapter, we solved this kind of problems by finding out the "speed" of decreasing the distance between two moving objects. After 1 hour John will move by 3.6 kilometers, Zoe will walk 2.4 kilometers, so the distance will decrease by $3.6 + 2.4 = 6$ kilometers, 6km per hour is the rate of distance reducing. This problem also can be seen as a problem of combined work of diminishing the distance. John need

$$12 : 3.6 = \frac{120}{36} = \frac{10}{3}\ hours = 3\frac{1}{3}\ hours$$

to cover the distance, so in 1 hour he will cover $\frac{3}{10}$ part of the distance.

Zoe needs

$$12 : 2.4 = \frac{120}{24} = 5\ hours$$

to cover the whole distance, and in 1 hour she will cover $\frac{1}{5}$ part. Both of them, walking toward each other will cover

$$\frac{3}{10} + \frac{1}{5} = \frac{3}{10} + \frac{2}{10} = \frac{5}{10} = \frac{1}{2}$$

of the distance, and definitely will need 2 hours to meet.

Exercises:

1. A moving walkway at an airport moves at a pace of 0.55 meters per second. If Peter stands on the walkway as it moves, how long will it take to transport him 200 meters? If he walks on this walkway at a speed of 4 km/h, how long will it take him to get to the end of the 200-meter-long walkway?

2. At the end of the walkway, Peter remembers he forgot his luggage at the security checkpoint. He turns around and runs at a speed of 10 km/h. How fast will he reach the starting point of the walkway (same 200-meter-long walkway)?

3. The speed of a boat in a still water on a lake is 12 km/h. The speed of the river flow is 3 km/h. How many hours does the boat need to go from the city A to the city B if the distance between the two cities is 45 km and the city A is up on the river, i.e. the river flows from A to B?

   How many hours does this boat need to go back from the city B to the city A?

4. The speed of the boat going downstream the river is 19 km/h, and the speed of the same boat going upstream this river is 15 km/h. What is the speed of the river stream and what is the speed of the boat in a still water on a lake?

5. A cat can eat the sausage in 10 minutes; a dog can eat it in 8 minutes. How fast will they eat the sausage together?

6. Mary, Peter, and Julia are going to do the spring cleanup in their garden. Mary can do the job in 4 hours, Peter can do the full clean up in 3 hours, Julia need 6 hours to do the job. How fast they will do it together?

7. A swimming pool can be filled by pump A in 3 hours and by pump B in 6 hours, each pump working on its own. At 9 am pump A is started. At what time will the swimming pool be filled if pump B is started at 10 am?

8. One tractor can plow a field in 15 days, while another can do it in 24 days. Which tractor will plow more: the first one in 8 days or the second one in 11 days?

9. The older brother can clean up the room in 2 hours, the younger brother can completely ruin it in 3 hours. In how many hours will the room be cleaned if they are locked together in the messy room? (it's a math problem, the answer "they will play games" will not be accepted!)

10. To bake 100 pancakes, Mom needs 30 minutes, while Dad needs 40 minutes. Son can eat 100 pancakes in an hour. Mom and Dad continuously make pancakes without stopping, while Son continuously eats them. After how much time from the beginning of this process will there be exactly 100 pancakes on the table?