

Coordinates.

Coordinates are a set of values that show an exact position. How many values do we need to show the exact position of a point on a number line? How many values do we need to find our place in a theater? In a plane? What we can use as values?

For example, the Johns family lives in Big Village, on Main Street, house number 33, NY, USA. To describe the location of Johns' house, we used several pieces of information, such as:

Country: USA,

State: NY,

Village: Big,

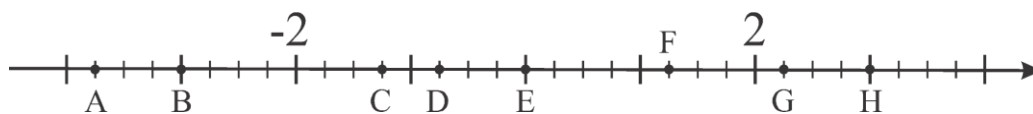
Street: Main,

House: 33.

On a number line, each point represents a number, and each number is linked to a point if an origin (the point at 0), a unit segment, and the positive direction are defined or can be defined based on the known information. This number is the coordinate of a point on the line in the defined system. The absolute value of this number tells us how many unit segments lie between this point and the origin, while the sign indicates on which side of the origin this point is located.

Example 1.

Find the coordinates of points A, B, C, D, E, F, G, and H on the number line below:



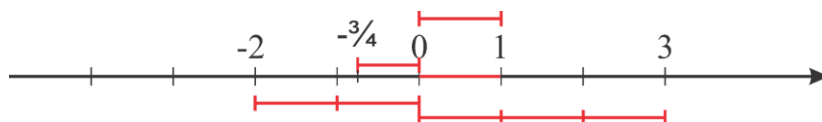
To find the coordinate of a point, we need to find how many unit segments can fit into the distance between 0 and the point, and then apply the corresponding sign, either plus or minus.

On this number line there are two known coordinates, -2 and 2 . Exactly 4 unit segments fit between two points, with 0 positioned exactly in the middle. The coordinate of point E is 0, $E(0)$.

The coordinate of the point F is 1 and $\frac{1}{4}$, $F(1\frac{1}{4})$.

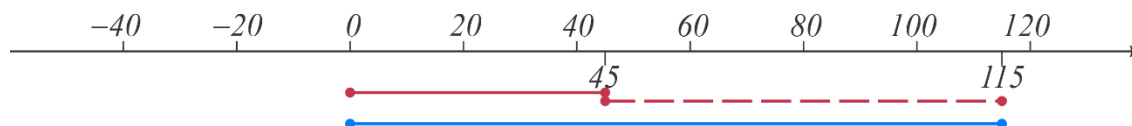
Each number has a property “absolute value”, it shows how far this number is from 0, the origin of the coordinate line. Absolute value of a number is always positive, the distance can't be expressed as a negative number. The formal definition of absolute value of any number is:

$$\begin{cases} |a| = a, & \text{if } a \geq 0 \\ |a| = -a, & \text{if } a < 0 \end{cases}$$



The addition and subtraction of positive and negative numbers along the coordinate axis.

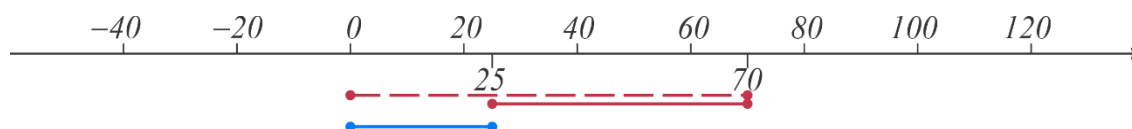
Two segments represent the absolute values of numbers 45, (solid line) and 70, (dotted line). When we need to add 45 and 70, we can sum their absolute values and the result will be positive. Addition of two positive numbers produces the positive result. The blue segment represents the absolute value of the result.



$$45 + 70 = 70 + 45 = |70| + |45| = 115$$

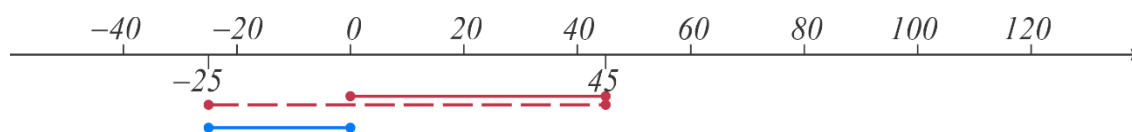
The result of subtraction of 45 from 70 is 25. The blue segment represents the absolute value of the result, segment with the length equal to the difference of two other segments.

$$70 - 45 = |70| - |45| = 25$$



The result of the subtraction of 70 from 45 will be the opposite of the result of $70 - 45$. The absolute value of the result will be the same (the difference between 70 and 45) but the sign would be “ - ”

$$45 - 70 = -(70 - 45) = -(|70| - |45|) = -25$$



So, if you need to subtract one number from the other: subtract the smaller number from the greater number and decide about the resulting sign.

Example 1:

$$25 - 77;$$

25 is smaller than 77, so the answer should be negative, the absolute value of the difference is the same as the absolute value of the difference of $77 - 25$; in other words, the result is a number, opposite to the difference $77 - 25$.

$$25 - 77 = -(77 - 25) = -(+52) = -52$$

Example 2:

$$168 - 230;$$

$168 < 230$, we can find the difference $230 - 168 = 62$ and write “-” before the resulting number.

$$168 - 230 = -(230 - 168) = -62$$

Let's take a look on the equation like

$$|x| = 12$$

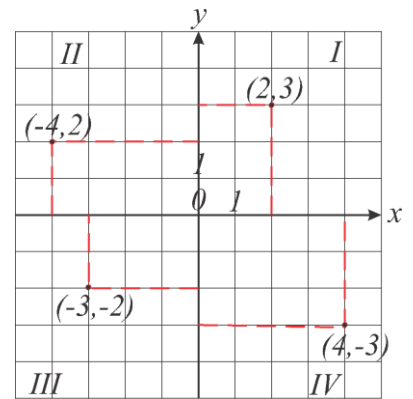
To which number x should be equal to make this equation true?

We are looking for a number which absolute value is 12. There are two such numbers, 12 and -12 . Therefore, this equation has two possible solutions.

To describe the position of the point on a plane, two numbers (parameters) are needed. For example, in a theater you need the row number and the number of your seat in the row. Or in an airplane people find their seats by the row number and letters for the seat: A, B, C, D ...

On a plane, each point corresponds to a unique ordered pair of numbers. To define these pairs, 2 perpendicular number lines are usually used. These two number lines intersect at the point called origin, associated with pair $(0,0)$, have the same unit segment, and are called axis, usually x and y axis.

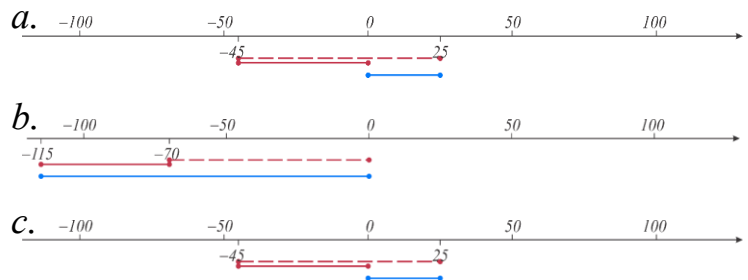
In this particular coordinate system, the two numbers allied with each point of the plane describe the distance from the point to both axes, and the signs of these numbers represent a quadrant where the point lies (quadrants I, II, III, and IV in the image above). Such a pair of numbers is an ordered pair, so the pair (n, m) and the pair (m, n) are linked to 2 different points. The absolute value of the first number in the pair is the distance to the y axis. The absolute value of the second one is the distance to the x axis.



Exercises:

1. Mark the points A(0), B(1), C($-1\frac{1}{2}$), D(5), E(-5), F(-3), G(3) on the number line in your notebook.

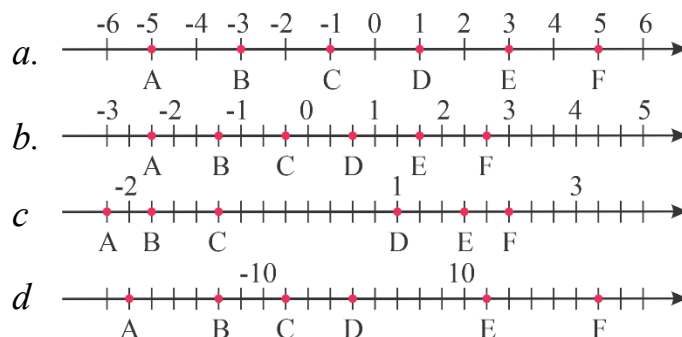
2. Write a problem, which can be represented by the drawing:



3. Evaluate: Example: $a. 7 + (-2) = 5$

- $a. (-10) + (+11);$ $b. (-7) + (-6);$ $c. (-4) + (+2);$
 $d. (-12) + (+3);$ $e. 15 - 18;$ $f. (-11) + (-20);$
 $g. 20 - 21;$ $h. (-100) + (-150);$ $i. (-3) + (+4);$

4. Write the coordinates of the points A, B, C, D, E, and F for the axes below:



5. On the axis points $A(-2)$ and $B(3)$ marked. Mark the origin (point is coordinate equal to 0) and unit segment. Find the coordinates of the points C, D, E.



6. Draw a coordinate system (only positive quadrant I). Create a picture by coordinates (connect points in the order):

$C_1(2, 0), C_2(2, 10), C_3(4, 12), C_4(12, 12), C_5(18, 14), C_6(18, 16), C_7(20, 14), C_8(22, 14), C_9(24, 12), C_{10}(24, 14), C_{11}(25, 12), C_{12}(26, 12), C_{13}(26, 14), C_{14}(28, 12), C_{15}(28, 10), C_{16}(24, 8), C_{17}(22, 8), C_{18}(18, 6), C_{19}(18, 0), C_{20}(14, 0), C_{21}(14, 4), C_{22}(6, 4), C_{23}(6, 0), C_1(2, 0).$