Math 4. Homework Exponents 2.

1. $a^n = \underbrace{a \cdot a \cdot a \dots \cdot a}_{n\ times}$
2. $a^n \cdot a^m = a^{n+m}$
3. $(a^n)^m = a^{n \cdot m}$
4. $a^1 = a$, for any a
5. $a^0 = 1$, for any $a \neq 0$
6. $(a \cdot b)^n = a^n \cdot b^n$

- A positive number raised into any power will result a positive number.
- A negative number, raised in a power, represented by an even number is positive, represented by an odd number is negative.

Exponent is very interesting mathematical operation. There is the story of the invention of the game of chess. The king ordered a new game because he was bored by the old games, was so happy about the new chess game that he said to the inventor: "*Name your reward and you will get it!*" The inventor asked for a simple reward. *"I would like to have one grain of rice on the first chess square, two on the second, four on the third and so on, doubling the amount of rice every square.*" The legend says that the King was surprised he didn't ask for gold but was quite content that the inventor asked for so little. But when the court scholars told him there wasn't enough rice in the whole world to fill the chess board, he had to admit his loss:

$1 + 2 + 2^2 + 2^4 + \cdots + 2^{63} = 18{,}446{,}744{,}073{,}709{,}551{,}615$

The weight of the rice grain is about 0.03g. so:

$$1.8 \cdot 10^{19} \cdot 0.03 = 5.4 \cdot 10^{17}\ g.\ or\ about\ 5.4 \cdot 10^{14} kg\ or\ 10^{15} lb.$$

Let's take a look on a fraction $\frac{27}{81}$:

$$\frac{27}{81} = \frac{3^3}{3^4} = \frac{3 \cdot 3 \cdot 3}{3 \cdot 3 \cdot 3 \cdot 3} = \frac{\cancel{3} \cdot \cancel{3} \cdot \cancel{3}}{\cancel{3} \cdot \cancel{3} \cdot \cancel{3} \cdot 3} = \frac{1}{3} = 3^{3-4} = 3^{-1}$$

$$\frac{1}{3} = 3^{-1}; \quad \frac{1}{3^3} = \frac{1}{3 \cdot 3 \cdot 3} = \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} = 1 : 3^1 : 3^1 : 3^1 = 1 : 3^3 = 3^{-3}$$

Negative power can be seen as a division exactly the same way as multiplication for the positive power.

$$\frac{a^n}{a^m} = \frac{\underbrace{a \cdot a \cdot \dots \cdot a}_{n\ times}}{\underbrace{a \cdot a \cdot \dots \cdot a}_{m\ times}} = \left(\underbrace{a \cdot a \cdot \dots \cdot a}_{n\ times}\right) : \left(\underbrace{a \cdot a \cdot \dots \cdot a}_{m\ times}\right) = a^n : a^m = a^{n-m}$$

Problem example.
What is greater 31^{11} or 17^{14}?

31 is less then 32, so 31^{11} is less than 32^{11}. But 32 is 2^5, therefore

$$32^{11} = (2^5)^{11} = 2^{5\cdot 11} = 2^{55}$$

And finally,

$$31^{11} < 32^{11} = (2^5)^{11} = 2^{5\cdot 11} = 2^{55} \text{ or } 31^{11} < 2^{55}$$

On the other hands, 17 is greater than 16, and $17^{14} > 16^{14}$; but 16 is 2^4, therefore

$$16^{14} = (2^4)^{14} = 2^{4\cdot 14} = 2^{56} \; and \; 17^{14} > 2^{56}$$

Finally, $31^{11} < 2^{55}$ and $17^{14} > 2^{56}$. 2^{55} is less than 2^{56}.

$$31^{11} < 2^{55} < 2^{56} < 17^{14}$$

$$31^{11} < 17^{14}$$

Homework

1. The number 64 can be represented in different ways as a power:

   $$64 = 2^6 = 4^3 = 8^2$$

   Write the following numbers in different ways as powers:

   $$a.\ 16, \quad b.\ 81, \quad c.\ 256, \quad d.\ 625, \quad e.\ 729, \quad f.\ 1000000$$

2. Suppose \$100 is deposited into an account and the amount doubles every 8 years. How much will be in the account after 40 years? Express your answer using powers.
3. Simplify (use exponent rules):

   a. $4^2 \cdot 4^2 \cdot 4^2 \cdot 4^2$; b. $(10^3)^5$; c. $(4c^2 \cdot c^3)^3$; d. $\frac{49^4 \cdot 7^5}{7^{12}}$

4. Compare the numbers (use >, <, =):

   a. 5^3 and $5 \cdot 3$; b. 127^{23} and 513^{18}; c. 2^{30} and 3^{20}