

Complex fractions.

Complex fractions are formed by two fractional and/or numeral expressions, one on the top and the other one at the bottom of the complex fraction, for example:

$$\frac{(2+3) \cdot 5}{7 - \frac{1}{2}}; \quad \frac{\frac{1}{2} + \frac{1}{3}}{\frac{7}{9} - \frac{2}{5}}$$

We know that the fraction bar is a just another way to write the division sign, so, the above expressions are equivalent to

$$\frac{(2+3) \cdot 5}{7 - \frac{1}{2}} = ((2+3) \cdot 5) : \left(7 - \frac{1}{2}\right); \quad \frac{\frac{1}{2} + \frac{1}{3}}{\frac{7}{9} - \frac{2}{5}} = \left(\frac{1}{2} + \frac{1}{3}\right) : \left(\frac{7}{9} - \frac{2}{5}\right)$$

It is easy to simplify a complex fraction:

$$\frac{\frac{1}{2} + \frac{1}{3}}{\frac{7}{9} - \frac{2}{5}} = \left(\frac{1}{2} + \frac{1}{3}\right) : \left(\frac{7}{9} - \frac{2}{5}\right) = \frac{\frac{3}{6} + \frac{2}{6}}{\frac{35}{45} - \frac{8}{45}} = \frac{\frac{5}{6}}{\frac{27}{45}} = \frac{5}{6} : \frac{27}{45} = \frac{5}{6} \cdot \frac{45}{27} = \frac{5}{1} \cdot \frac{5}{3} = \frac{25}{3}$$

Now let's simplify a little more sophisticated complex fraction:

Example 1.

$$\frac{1\frac{2}{3} + 2\frac{4}{9}}{4\frac{26}{27} - 2\frac{2}{9}} = \left(1\frac{2}{3} + 2\frac{4}{9}\right) : \left(4\frac{26}{27} - 2\frac{2}{9}\right)$$

$$1\frac{2}{3} + 2\frac{4}{9} = 1 + \frac{2}{3} + 2 + \frac{4}{9} = 3 + \frac{6}{9} + \frac{4}{9} = 3 + \frac{10}{9} = \frac{27}{9} + \frac{10}{9} = \frac{37}{9}$$

$$4\frac{26}{27} - 2\frac{2}{9} = 4 + \frac{26}{27} - 2 - \frac{2}{9} = 2 + \frac{26}{27} - \frac{6}{27} = 2 + \frac{20}{27} = \frac{54}{27} + \frac{20}{27} = \frac{74}{27}$$

$$\frac{37}{9} : \frac{74}{27} = \frac{37}{9} \cdot \frac{27}{74} = \frac{37 \cdot 3 \cdot 9}{9 \cdot 2 \cdot 37} = \frac{3}{2} = 1\frac{1}{2}$$

Example 2:

$$\frac{3\frac{4}{7} : \left(6\frac{1}{28} - 3\frac{3}{4}\right)}{\left(1\frac{5}{6} \cdot 1\frac{5}{22}\right) : 18 \cdot 5}$$

First, let's find the value of the numerator:

$$3\frac{4}{7} : \left(6\frac{1}{28} - 3\frac{3}{4}\right) = \frac{25}{7} : \left(5 + 1\frac{1}{28} - 3 - \frac{3}{4}\right) = \frac{25}{7} : \left(2 + \frac{29}{28} - \frac{3}{4}\right) = \frac{25}{7} : \left(2 + \frac{29}{28} - \frac{21}{28}\right) \\ = \frac{25}{7} : \left(2\frac{8}{28}\right) = \frac{25}{7} : \frac{64}{28} = \frac{25}{7} \cdot \frac{28}{64} = \frac{25 \cdot 4}{64} = \frac{25}{16}$$

The value of the denominator is

$$\left(1\frac{5}{6} \cdot 1\frac{5}{22}\right) : 18 \cdot 5 = \left(\frac{11}{6} \cdot \frac{27}{22}\right) \cdot \frac{1}{18} \cdot 5 = \frac{11 \cdot 3 \cdot 9}{2 \cdot 3 \cdot 11 \cdot 2} \cdot \frac{1}{2 \cdot 9} \cdot 5 = \frac{5}{8}$$

Finally:

$$\frac{\frac{25}{16}}{\frac{5}{8}} = \frac{25}{16} : \frac{5}{8} = \frac{25}{16} \cdot \frac{8}{5} = \frac{5 \cdot 5 \cdot 8}{2 \cdot 8 \cdot 5} = \frac{5}{2} = 2\frac{1}{2}$$

In expressions where the fraction bar is used multiple times, the ' = ' symbol is placed at the bar representing the last division operation in order:

$$\frac{3}{\frac{3}{7}} = 3 : \frac{3}{7} = 3 \cdot \frac{7}{3} = 7; \quad \frac{\frac{3}{7}}{7} = \frac{3}{7} : 7 = \frac{3}{7} \cdot \frac{1}{7} = \frac{3}{14}$$

Exercises:

1. Write the expressions as fractions and evaluate:

a. $14:42$; b. $2:3:5$; c. $2:8:3$; d. $100 \cdot 6:40$; e. $5:15 \cdot 3$

f. $(21 \cdot 18):14$; g. $50:(16 \cdot 25)$; h. $(12 \cdot 15):40$; i. $(4 \cdot 24):(2 \cdot 8)$

2. Write the expressions as fractions and evaluate:

Example:

$$(3 \cdot 5 \cdot 5 \cdot 5 \cdot 7):(5 \cdot 7) = \frac{3 \cdot 5 \cdot 5 \cdot 5 \cdot 7}{5 \cdot 7} = 3 \cdot 5 \cdot 5 = 75$$

a. $(3 \cdot 3 \cdot 5 \cdot 11):(3 \cdot 11)$; b. $(2 \cdot 2 \cdot 3 \cdot 5 \cdot 7):(2 \cdot 3 \cdot 7)$;

c. $(2 \cdot 3 \cdot 7 \cdot 13):(3 \cdot 7)$; d. $(3 \cdot 5 \cdot 11 \cdot 17 \cdot 23):(3 \cdot 11 \cdot 17)$;

3. Evaluate:

a. $\frac{5}{\frac{2}{3}}$; b. $\frac{\frac{5}{2}}{3}$; c. $\frac{\frac{3}{4}}{\frac{9}{16}}$; d. $\frac{\frac{5}{17}}{\frac{15}{34}}$;

4. Evaluate:

$$\begin{array}{lllll}
 a. \frac{6}{1 - \frac{1}{3}}; & b. \frac{1 - \frac{1}{6}}{2 + \frac{1}{6}}; & c. \frac{\frac{1}{2} + \frac{3}{4}}{\frac{1}{2}}; & d. \frac{\frac{7}{10} + \frac{1}{3}}{\frac{7}{10} + \frac{1}{2}}; & e. \frac{2 - \frac{\frac{1}{2} - \frac{1}{4}}{2}}{2 + \frac{\frac{1}{2} - \frac{1}{4}}{2}} \\
 f. \frac{\frac{1}{2} - \frac{1}{6}}{2}; & g. \frac{\frac{9}{16}}{\frac{3}{4} - \frac{3}{8}}; & h. \frac{\frac{5}{6} - \frac{2}{3}}{\frac{1}{3}}; & i. \frac{\frac{1}{2} + \frac{1}{6}}{\frac{1}{2} - \frac{1}{6}}; & j. \frac{\frac{17}{14} - \frac{5}{7}}{\frac{11}{21} + \frac{1}{7}}
 \end{array}$$

5. A number was decreased by $\frac{3}{10}$ of itself, and the result was 210. Find the number.

6. A number was increased by $\frac{1}{7}$ of itself, and the result was 56. What is the number?

7. Evaluate (answer is $\frac{9}{11}$):

$$\frac{3\frac{5}{11} \cdot 6\frac{3}{4}}{3\frac{5}{11} \cdot 6\frac{3}{4} + 3\frac{5}{11} \cdot 1\frac{1}{2}}$$

8. Evaluate (answer is $\frac{25}{39}$):

$$\frac{(\frac{1}{6} + \frac{1}{10} + \frac{1}{15}) : (\frac{1}{6} + \frac{1}{30})}{(\frac{1}{6} + \frac{1}{4} - \frac{1}{5}) : (\frac{1}{4} - \frac{1}{6})}$$

9. Are these two expressions equivalent?

$$1 - \frac{1}{5} : \frac{1}{5} \quad \text{and} \quad \frac{1 - \frac{1}{5}}{\frac{1}{5}}$$

Evaluate both expressions.

10. Into how many equal $\frac{1}{5}$ kilogram portions 5 kg cake can be divided?

11. The area of a rectangle is $\frac{5}{7} m^2$. One side of this rectangle is $\frac{3}{4} m$. What is the length of the other side?

12. Julia and Mary ate all the Halloween candy. Mary claimed that she ate $\frac{2}{3}$ of the candies, and Julia said that she ate $\frac{3}{5}$. Their parents think that something is wrong. Are they right?