Math 4b. Class work 6.

Multiplication of a fraction by a number.

To multiply a fraction by a number, simply multiply the numerator by the number:

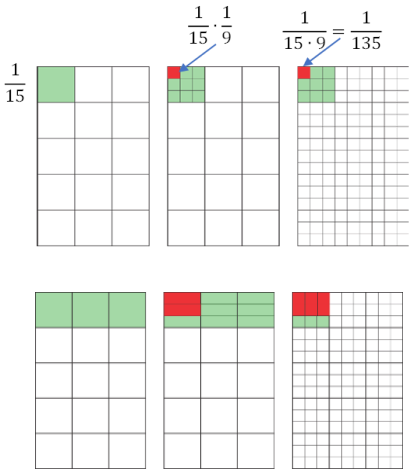
$$\frac{2}{7}\cdot 3=\frac{2}{7}+\frac{2}{7}+\frac{2}{7}=\frac{2+2+2}{7}=\frac{3\cdot 2}{7}=\frac{6}{7}$$

On the other hand:

$$\frac{2}{7}\cdot 3=3\cdot\frac{2}{7}=3:7\cdot 2=(3:7)\cdot 2=(3\cdot 2):7=\frac{3\cdot 2}{7}$$

Multiplication of a fraction by a fraction.

$\frac{1}{15}$ is one part of a whole divided into 15 equal small parts.



If we want to take $\frac{1}{9}$ part of this little $\frac{1}{15}$ chunk, we have to divide it into 9 even smaller pieces, to find $\frac{1}{9}$th of $\frac{1}{15}$th.

$$\frac{1}{15}:9=\frac{1}{15}\cdot\frac{1}{9}=\frac{1}{15\cdot 9}=\frac{1}{135}$$

If we need to take two small $\frac{1}{9}$ of $\frac{1}{15}$

$$\frac{1}{15}:9\cdot 2=\frac{1}{15}\cdot\frac{2}{9}=\frac{1\cdot 2}{15\cdot 9}=\frac{2}{135}$$

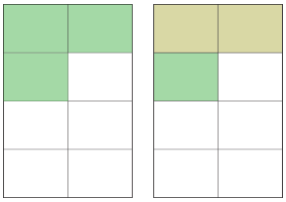
Or, if we want to find out $\frac{2}{9}$ of $\frac{3}{15}$.

$$\frac{3}{15}:9\cdot 2=\frac{3}{15}\cdot\frac{2}{9}=\frac{3\cdot 2}{15\cdot 9}=\frac{6}{135}$$

To multiply two fractions, we need to multiply numerators, multiply denominators and reduce fraction, if possible. The easiest way to do it, first (prime) factorize all the numerators and denominators, divide both part of the fraction by the same numbers and multiply the rest
Examples:

$$\frac{26}{27}\cdot\frac{21}{52}=\frac{13\cdot 2}{3\cdot 3\cdot 3}\cdot\frac{7\cdot 3}{13\cdot 4}=\frac{\cancel{13}\cdot\cancel{2}\cdot 7\cdot\cancel{3}}{\cancel{3}\cdot 3\cdot 3\cdot\cancel{13}\cdot\cancel{2}\cdot 2}=\frac{7}{3\cdot 3\cdot 2}=\frac{7}{18}$$

Division of fractions.



More of multiplication of fractions:

$$\frac{3}{8}\cdot\frac{2}{3}=\frac{2}{8}=\frac{1}{4}$$

Remember, when we divide a number by another number, we a looking for the answer, that will give us a dividend when multiplied by divisor.

Based on this definition, from the example above one can see that division of $\frac{1}{4}$ by $\frac{2}{3}$ should give the quotient $\frac{3}{8}$, and division of $\frac{1}{4}$ by $\frac{3}{8}$ should result as $\frac{2}{3}$.

$$\frac{1}{4}:\frac{2}{3}=\frac{3}{8}$$

We can notice that the multiplication of $\frac{1}{4}$ by the inverse fraction $\frac{3}{2}$ will produce exactly $\frac{3}{8}$;

$$\frac{1}{4}:\frac{2}{3}=\frac{1}{4}\cdot\frac{3}{2}=\frac{3}{8}$$

To divide one fraction by another we need to multiply the dividend by the inverse (reciprocal) fraction. Two fractions are inverse fractions if their product is 1. Inverse fractions can also be called reciprocal.

Two fractions are called *inverse fractions* if their product is equal to 1.

Examples:

$$\frac{1}{4}\cdot\frac{4}{1}=1;\qquad \frac{3}{5}\cdot\frac{5}{3}=1;\qquad \frac{4}{7}\cdot\frac{7}{4}=1;$$

Division can be seen as multiplication by a reciprocal number, for example:

$$3:5=3\cdot\frac{1}{5}=\frac{3}{5};\qquad \frac{7}{9}:7=\frac{7}{9}\cdot\frac{1}{7}=\frac{1}{9};$$

Among two reciprocal fractions, one is a proper fraction and another is improper fraction.

The best and easiest way is multiply (divide) mixed numbers, is to rewrite them as improper fractions and then just multiply these fractions, or, in case of division, multiply on improper fraction and a fraction, reciprocal to the divisor.

For example:

$$1\frac{3}{4}\cdot 2\frac{2}{5}=\frac{7}{4}\cdot\frac{12}{5}=\frac{7\cdot 3\cdot 4}{4\cdot 5}=\frac{21}{5}=4\frac{1}{5}$$

Exercise:

1. Write division operation as fraction, reduce fraction, if possible:

   Example:

$$4:8=\frac{4}{8}=\frac{1\cdot 4}{2\cdot 4}=\frac{1}{2}$$

$a.\ 3:7;$ $b.\ 5:15;$ $c.\ 3:9;$ $d.\ 4:9;$ $e.\ 12:13.$

2. Write fraction as a division, reduce fraction if possible:

   Example:

$$\frac{3}{12}=\frac{1}{4}=1:4$$

$a.\ \frac{4}{5};$ $b.\ \frac{7}{9};$ $c.\ \frac{5}{15};$ $d.\ \frac{2}{8};$ $e.\ \frac{11}{44}.$

3. Evaluate:

$a.\ \frac{4}{5}\cdot\frac{5}{7};$ $b.\frac{2}{3}:\frac{5}{7};$ $c.\frac{8}{9}\cdot\frac{3}{5};$ $d.\ \frac{1}{4}:\frac{1}{2};$ $e.\ \frac{9}{2}\cdot\frac{2}{9};$

$f.\frac{8}{21}\cdot\frac{7}{10};$ $g.\frac{3}{4}:\frac{1}{2};$ $h.\frac{8}{15}\cdot\frac{25}{28};$ $i.\ \frac{5}{6}:\frac{7}{12};$ $j.\ \frac{4}{9}:\frac{8}{9};$

4. Painter painted $\frac{2}{7}$ of the house in 4 days. How many days will it take him to paint the whole house?

5. Evaluate:

$a.\ \frac{3}{7}\cdot 2;$ $b.3\cdot\frac{1}{6};$ $c.\ 9\cdot\frac{5}{6};$ $d.\ \frac{2}{3}\cdot 2;$ $e.4\cdot\frac{1}{2};$

6. Evaluate:

$a.\ \frac{1}{3}\cdot\frac{2}{7};$ $b.\ \frac{1}{2}\cdot\frac{5}{6};$ $c.\ \frac{1}{2}\cdot\frac{1}{3};$ $d.\ \frac{3}{5}\cdot\frac{1}{2}\cdot\frac{4}{9};$ $e.\ \frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}\cdot\frac{4}{5}$

$f.\ \frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}\cdot\frac{4}{5}\cdot\ldots\cdot\frac{23}{24}\cdot\frac{24}{25}$

7. Write the fraction inverse to the given fraction:

   Example:

$$\frac{3}{4}\rightarrow\frac{4}{3};\quad\frac{3}{4}\cdot\frac{4}{3}=1$$

$a.\ \frac{3}{7};$ $b.\frac{7}{9};$ $c.\ \frac{12}{5};$ $d.\ \frac{9}{4};$ $e.\frac{1}{3};$ $f.\ \frac{1}{3};$ $g.\ \frac{m}{n}\ (m,n\neq 0);$

8. Evaluate:

$a.\ \frac{2}{3}:\frac{5}{7};$ $b.\ \frac{1}{4}:\frac{1}{2};$ $c.\ \frac{3}{4}:\frac{1}{2};$ $d.\ \frac{4}{9}:\frac{8}{9};$ $e.\ \frac{5}{6}:\frac{7}{12};$

$f.\ 2:\frac{1}{7};$ $g.\ 4:\frac{2}{3};$ $h.\ 3:\frac{1}{2};$ $i.\ 1:\frac{2}{7};$ $j.\ 1:\frac{1}{4};$

9. Evaluate:

$a.\ 14:42;$ $b.\ 2:3:5;$ $c.\ 2:8\cdot 3;$ $d.\ 100\cdot 6:40;$ $e.\ 5:15\cdot 3.$

10. Evaluate:

Example:

$$3\frac{1}{4}\cdot 2=\left(3+\frac{1}{4}\right)\cdot 2=3\cdot 2+\frac{1}{4}\cdot 2=6+\frac{1\cdot 2}{4}=6+\frac{2}{4}=6+\frac{1}{2}=6\frac{1}{2}$$

Or it can be done by transferring the mixed number into the improper fraction:

$$3\frac{1}{4}\cdot 2=\frac{3\cdot 4+1}{4}\cdot 2=\frac{13}{4}\cdot 2=\frac{13\cdot 2}{2\cdot 2}=\frac{13}{2}=\frac{12+1}{2}=6+\frac{1}{2}=6\frac{1}{2}$$

$a.\ 2\frac{1}{3}\cdot 2;$ $b.\ 4\cdot 1\frac{1}{2};$ $c.\ 1\frac{1}{3}\cdot 9;$ $d.\ \frac{3}{7}\cdot 2\frac{1}{3};$ $e.\ 1\frac{1}{3}\cdot 1\frac{2}{2}$

$f.\ 12\cdot\frac{1}{6}\cdot 1\frac{1}{2}\cdot 3\frac{3}{4}\cdot 4\frac{1}{5};$ $g.\ 3\cdot 5\frac{1}{4}\cdot 1\frac{1}{7}\cdot 5\frac{1}{2}\cdot\frac{4}{11};$

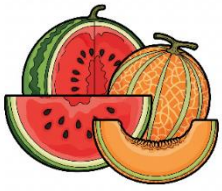
11. Evaluate:

$a.\ \frac{4}{7}\cdot\frac{5}{24}:1\frac{1}{14};$ $b.\ 25\cdot\frac{7}{15}:\frac{7}{9};$ $c.\ \frac{7}{18}:\frac{20}{21}:\frac{5}{12};$ $d.\ \frac{5}{9}\cdot 2\frac{1}{4}:20;$

12. Find the unknown:

$a.\ \frac{1}{3}\cdot x=\frac{1}{6};$ $b.\ \frac{2}{5}\cdot x=\frac{1}{5};$ $c.\ \frac{2}{3}\cdot x=1$

$d.\ x\cdot 6=\frac{5}{6};$ $e.\ x\cdot 6=4;$ $f.\ 3\cdot x=\frac{1}{3}$



13. Melon weighs 7 pounds, and the watermelon is $1\frac{1}{5}$ times heavier. How many pounds watermelon is heavier than the melon?