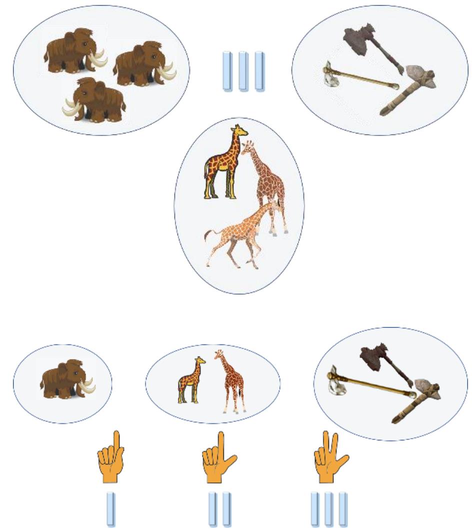
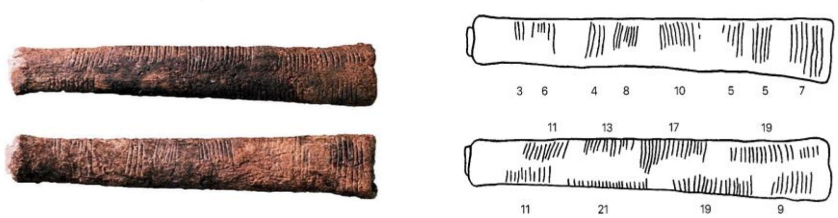
Math 4b. Class work 2(a).

A very long time ago, our ancestors realized that each group of objects possesses a quantitative property — the number of objects in the group. This property does not depend on the nature of the objects themselves. Groups can be compared based on this property, determining which group has more and which group has fewer items. This gave rise to the concept of "number" (now we use the term "*natural numbers*" for numbers to count objects).



Initially, prehistoric people compared the number of objects in the group with the count of fingers– since hands were ever-present! Then they began scratching marks on wood and bones as a means to record quantities. One of the oldest known examples of such bones is the Ishango bone, which dates back to around 20,000-25,000 years ago.

Historians continue to debate the purpose of the Ishango bone. Some think that might have been an early calculating tool due to its tally marks grouped in a specific manner. (Of course, we cannot possibly know how exactly it was used.)



The next step in the progress of math is the creation of the arithmetic operations. What can we do with numbers? Numbers can be added together. At first, the operation +1 was developed:

||| + | = ||||

(There were no "+" sign, but we can use it for convenience.)
Then addition of two groups made the big progress:

||||||| + ||| = ||||||||||

$3 + 5 = 8$ (3 – *addend*, 5 – *addend*, 8 – *sum*)

$8 - 3 = 5$ (8 – *minuend*, 3 – *subtrahend*, 5 – *difference*)

$8 - 5 = 3$ (8 – *minuend*, 5 – *subtrahend*, 3 – *difference*)

All the numbers we now use to count are called natural numbers. In the later times, various systems of writing numbers were developed, and in our present decimal system we can write the same as:

$$3 + 5 = 8$$

The operation of subtraction for natural numbers is the way to find the number that, when added to the numbers we are subtracting, results in the initial number. There are special names for the numbers in this operation:

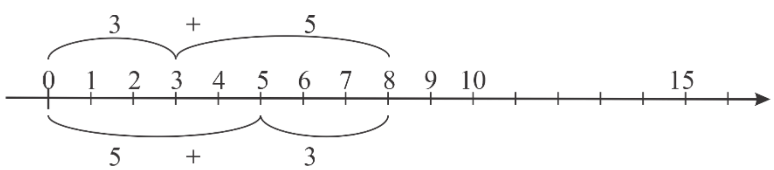
$$addend + addend = sum$$

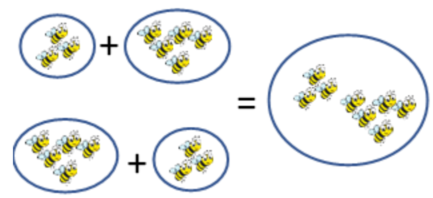
$$minuend - subtrahend = difference$$

So, if we add subtrahend to a difference, the result should be the minuend.
The operation of addition has a few properties:

- it's commutative: it does not matter what addend goes first; the sum will not change.
- it's associative: if three terms are added together, it doesn't matter how they are added—whether the first two and then the third or the second plus the third and then the first—the result will remain the same.

Commutative and associative properties of addition are intuitively easy to understand.





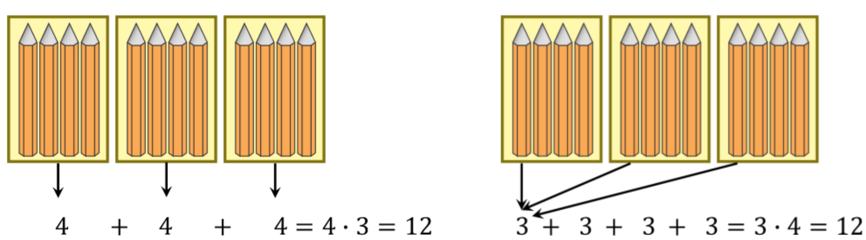
After that, multiplication was introduced as addition of the same addend (term) several times.

$$4 \cdot 3 = \underbrace{4 + 4 + 4}_{3\ times} = \underbrace{3 + 3 + 3 + 3}_{4\ times} = 12$$

We can write it using variables, in a general form:

$$c \cdot b = \underbrace{c + c + \cdots + c}_{b\ times} = \underbrace{b + b + \cdots + b}_{c\ times} = a$$

The result of multiplication is called product, and the participants of the operation are called factors. $\boldsymbol{c}$ and $\boldsymbol{b}$ are factors, and $\boldsymbol{a}$ is a product.

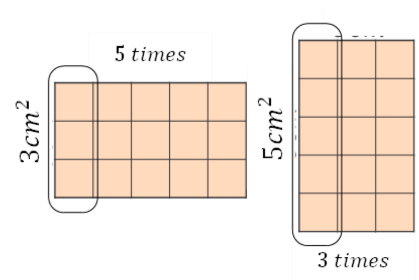


How many pencils are there in three boxes, if there are four pencils in each box?
$3 \cdot 4 = 12$; three and four are factors, 12 is a product.

Multiplication also has properties:

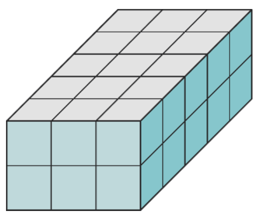
- it's commutative. It does not matter what factor goes first; the product will not change.

- it's associative. If three factors are multiplied, it doesn't matter whether we first get the product of the first two and then multiply by the third factor or the second and third multiplied first, and then the result is multiplied by the first factor—the product will still be the same.



The commutative property of multiplication can be also illustrated by calculating the area of a rectangle:

$$S = 3cm^2 \cdot 5times = 5cm^2 \cdot 3times = 15cm^2$$



The associative property can be shown by calculating the volume of parallelepiped. How many cubed are stacked together into the parallelepiped?
First, we can multiply five by three to find out how many cubes are in the horizontal slice, and then multiply by 2 (the number of slices). Or multiplication of three by two will provide the number of cubes on the front slice, and there are five such slices.

$$(3 \cdot 5) \cdot 2 = (2 \cdot 5) \cdot 3 = (2 \cdot 3) \cdot 5 = 2 \cdot 3 \cdot 5 = 24$$

	Commutative	Associative	Identity
Addition	$a + b = b + a$	$(a + b) + c = a + (b + c)$	$a + 0 = a$
Multiplication	$a \cdot b = b \cdot a$	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$	$a \cdot 1 = a;$ $a \cdot 0 = 0$
Distributive property of multiplication over addition	$a \cdot (b + c) = a \cdot b + a \cdot c$		

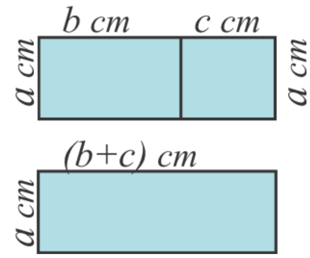
Distributive property can be illustrated with the following problem:
The farmer put green and red grapes into boxes. Each box contains 5lb of grapes. How many pounds of green and red grapes altogether did the farmer put into boxes if he had 10 boxes of green and 8 boxes of red grapes?

We can first find out how many boxes of grapes the farmer has altogether and multiply it by 5lb in each box, or we can find out the weight of white and red grapes in the boxes and then add it.

$$5 \cdot (10 + 8) \quad or \quad 5 \cdot 10 + 5 \cdot 8$$

$$5 \cdot 18 = 50 + 40; \qquad 90 = 90$$

Another example:



The combined area of these two rectangles is the sum of two areas, $S = a \cdot b + a \cdot c$

The rectangle with one side $a\ cm$ and the other side $(b + c)$ cm will have exactly the same area, $S = a \cdot b + a \cdot c = a \cdot (b + c)$. (I have used the variables a, b, and c instead of numbers to show the distribution property in a general way).

Using the distributive property, we can factor out the common factor of two terms of the expression, for example:

$$6 \cdot 7 + 6 \cdot 3 = 6(7 + 3) = 6 \cdot 10 = 60$$

Properties of the arithmetic operations can be used to simplify the calculations.

Example:

We need to multiply a two-digit number by a single digit number.

$28 \cdot 9 = (20 + 8) \cdot 9 = 20 \cdot 9 + 8 \cdot 9 = 180 + 72$

$180 + 72 = 100 + 80 + 70 + 2 = 100 + 150 + 2 = 252$

Another example: how to find the value of the expression: $250 \cdot 61 - 25 \cdot 390$

Of course, it can be calculated in a very straightforward way:

$250 \cdot 61 - 25 \cdot 390 = 15250 - 9750 = 5500$

Or we can use the properties of multiplication (which property has been used here?):

$25 \cdot 390 = 25 \cdot 39 \cdot 10 = 250 \cdot 39$

$250 \cdot 61 - 25 \cdot 390 = 250 \cdot 61 - 250 \cdot 39 = 250(61 - 39) = 250 \cdot 22 =$

$= 25 \cdot 22 \cdot 10 = (20 + 2) \cdot 25 \cdot 10 = (20 \cdot 25 + 2 \cdot 25) \cdot 10 = 550 \cdot 10 = 5500;$

Exercises:

1. Do the calculations in your head with the help of the properties of arithmetic operations:

   $25 \cdot 8; \quad 132 + 221; \quad 248 - 134; \quad 9 \cdot 38; \quad 321 \cdot 41 + 59 \cdot 321;$

2. Using the equality, $678 + 1357 = 2035$ find the value of

   $a.\ 2035 - 1357; \qquad b.\ 2035 - 678;$

3. Fill the empty boxes in the table (draw the table in your notebook):

a	16	28	44		49			
b	7	13		18		12	23	
$a+b$			60	67			833	72
$a-b$					35	60		0

4. In the number 3,728,954,106, remove three digits so that the remaining digits in the same order represent
   a. the smallest seven-digit number;
   b. the largest seven-digit number;

5. List all digits that can be written in place of the asterisk so that the resulting inequality is true:

   $a. 7*38 > 7238$ $\quad b. 96*4 < 9614$ $\quad c. 1596 > 159*$ $\quad d. 478* > 4783$

6. Write**:**

   a. the smallest even 10-digit number in which all digits are different;
   b. the largest odd 10-digit number in which all digits are different.

7. Using the given equation, compose two more:
   Example: From the equation $3415 + 767 = 4182$ it follows that $4182 - 767 = 3415$, and also that $4182 - 3415 = 767$
   $a.\ 784 + 57 = 841;$ $\quad b.\ 1234 - 342 = 892;$
   $c.\ 3256 + 1254 = 4510;$ $\quad d.\ 4325 - 1123 = 3202$

8. Find the unknown:
   $a.\ b + 1111 = 3000;$ $\quad b.\ 456 + c = 1362;$ $\quad c.\ p + 222 = 152;$

   $d.\ 1765 - n = 753;$ $\quad e.\ a - 56 = 135;$ $\quad f.\ l - 175 = 1889;$

   $g.\ 2050 - f = 12;$ $\quad h.\ 345 + m = 1214;$

9. Write without parenthesis and evaluate, also evaluate with parenthesis:
   Example: $123 - (12 + 5) = 123 - 12 - 5 = 106$
   $123 - (12 + 5) = 123 - 17 = 106$

   $a.\ 234 + (34 - 12);$ $\quad b.\ 432 - (35 + 230);$ $\quad c.\ 527 - (78 - 23).$

   $d.\ 374 - (15 + 22);$ $\quad e.\ 134 + (251 - 230);$ $\quad f.\ 651 - (65 - 12).$

10. Evaluate (what is the best way to compute it? Hint: use commutative property):
Example: $3 \cdot 2 \cdot 7 \cdot 8 \cdot 5 = 2 \cdot 5 \cdot 3 \cdot 7 \cdot 8 = 10 \cdot 21 \cdot 8 = 10 \cdot 194 = 1680$

$a.\ 72 + 59 + 97 + 28 + 41;$ $b.\ 32 + 34 + 36 + 38;$
$c.\ 5 \cdot 19 \cdot 5 \cdot 3 \cdot 2 \cdot 2;$ $d.\ 715 - 99 - 101;$
$e.\ (629 + 56) - 629;$ $e. 232 - (95 + 132);$

11. Evaluate by the most convenient way:
$a.\ 894 - (294 + 80);$ $b.\ (586 + 245) - 486;$ $c.\ (324 + 498) - 298$

12. Solve the problems in your head, in your notebook write only answers.
$a.\ 68 + 45 + 32;$ $b.\ 437 + 39 + 13;$ $c.\ 784 + 79 + 21;$
$d.\ 122 + (73 + 58);$ $e.\ 43 + 96 + 57;$ $f.\ 144 + (56 + 99);$

13. Solve the problems in your head, in your notebook write only answers.
$a.\ 3 \cdot 2 \cdot 5$ $b.\ 7 \cdot 25 \cdot 4;$ $c.\ 2 \cdot 7 \cdot 5;$ $d.\ 125 \cdot 7 \cdot 8$
$e.\ 16 \cdot 25 \cdot 4$ $f.\ 4 \cdot 9 \cdot 25;$ $g.\ 12 \cdot 8 \cdot 125;$ $h.\ 13 \cdot 2 \cdot 5$

14. Write without parenthesis, use the distributive property:
Example: $21 \cdot (2 + 3) = 21 \cdot 2 + 21 \cdot 3$
$a. 32 \cdot (21 + 3);$ $b.\ 5 \cdot (21 - 3);$ $c.\ 7 \cdot (12 + 5);$ $d.\ 3 \cdot (45 + 15);$

15. Evaluate (what is the best way to compute it? Hint: use the distributive and/or commutative properties):
Example: $17 \cdot 55 + 45 \cdot 17 = 17 \cdot (55 + 45) = 17 \cdot 100 = 1700$

$a.\ 23 \cdot 15 + 15 \cdot 77;$ $b.\ 79 \cdot 21 - 69 \cdot 21;$

$c.\ 340 \cdot 7 + 16 \cdot 70;$ $d.\ 250 \cdot 61 - 25 \cdot 390;$

$e.\ 67 \cdot 58 + 33 \cdot 58;$ $f.\ 55 \cdot 682 - 45 \cdot 682$

$g.\ 5 \cdot 4 + 5 \cdot 44 + 5 \cdot 444 + 5 \cdot 4444;$

$$h.\ 26 \cdot 25 - 25 \cdot 24 + 24 \cdot 23 - 23 \cdot 22 + 22 \cdot 21 - 21 \cdot 20 + 20 \cdot 19 - 19 \cdot 18 + 18 \cdot 17 - 17 \cdot 16 + 16 \cdot 15 - 15 \cdot 14;$$

16. Using the commutative and associative properties of addition or multiplication simplify the expressions:
Example: $43 + b + 15 = 43 + 15 + b = 58 + b$
$a.\ 23 + a + 67;$ $b.\ 42 + b + 34 + 128;$

$c.\ 15 \cdot c \cdot 4;$ $d.\ 2 \cdot d \cdot 7 \cdot 5 \cdot 5 \cdot 2$

17. Compare the numbers with missing digits, if possible. If it's not possible, explain why:

$a.\ 9**\ \ldots 2**$ $\quad b.\ 18***\ \ldots 20***$ $\quad c.\ 3***4 \ldots 3***7$

$d.\ 6**** \ldots 6*5**$ $\quad e.\ 9*4*4 \ldots 8*4*4$ $\quad f.\ **111 \ldots *1111$

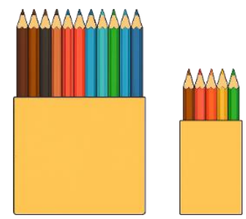
18. Compare without doing actual calculations (put <, >, or =):

$a.\ 2453+235\ \ldots\ 2453+236$ $\quad b.\ 234\cdot 123\ \ldots\ 234\cdot 122$

$c.\ 2341-123\ \ldots\ 2341-122$ $\quad d.\ 456\div 4\ \ldots\ 456\div 3$

$e.\ a\div 4\ \ldots\ a\div 3$ $\quad f.\ b+235\ \ldots\ b+236$

19. Solve the problems:
   a. A book has 256 pages. Andrew has read 192 pages. How many times is the number of pages he has read greater than the number of pages he hasn't read?
   b. The distance between two cities is 420 km. The car has traveled 84 km and then stopped. How many times is the distance he has traveled less than the remaining distance?



20. 8 large boxes contain the same number of pencils as 12 small boxes. How many pencils are in one small box if one large box contains 18 pencils?

21. A garland is made of red, blue, green, and yellow lights. The total number of red, blue, and green lights is 37. The total number of blue, green, and yellow lights is 29. The total number of red, green, and yellow lights is 32. There are 44 lights in total on the garland. How many lights of each color are there?

22. Find missing digits in the problems:

$$\begin{array}{r} \square 9\,5 \\ +\ 3\,\square 4 \\ \hline 8\,4\,\square \end{array} \qquad \begin{array}{r} \square 2\,0 \\ -\ 4\,\square 7 \\ \hline 3\,6\,\square \end{array}$$

23. Fill the empty spaces in the magic squares, so that all the columns, rows, and diagonals have the same sum of numbers.

a.

2		6
	5	1
4		

b.

3		15	14
13	16		
10	11		
8		12	9

24. A farmer prepared 12 liters of strawberry jam for the winter, raspberry jam — 4 liters less than strawberry, and apple jam — 2 times more than strawberry and raspberry together. How many liters of jam did the farmer prepare in total for the winter?

25. One book has 126 pages, and the other has 84 pages. Michle read both books in 5 hours. How much time did he spend reading each book if his reading rate did not change?

26. There are 160 notebooks in two boxes. In one box, there are 20 notebooks more than in the other. How many notebooks are in each box?

27. Alice was counting the steps of a staircase. Between the fifth and first floors, she counted 100 steps. How many steps are there between the first and second floors if the number of steps between all floors is the same?

28. In the expression place parentheses so that its value equals 10.

$$5 \cdot 8 + 12 \div 4 - 3$$

29. Find two consecutive natural numbers whose sum is 1001.

30. Create a problem, that can be solved by the following steps, write the solution as a single numeral expression:

1. $12 + 5 = 17$ 2. $12 - 4 = 8$ 3. $12 + 17 + 8 = 37$