

USEFUL RESOURCES

The updates, homework assignments, and useful links for APC can be found on SchoolNova's web page:

[https://schoolnova.org/classinfo?class\\_id=2252&sem\\_id=74](https://schoolnova.org/classinfo?class_id=2252&sem_id=74)

The practical information about the club and contacts can be found on the same web page.

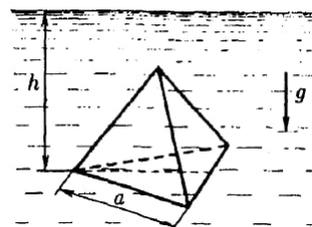
TODAY'S MEETING

We solved most of the assigned problems on hydrostatics. The remaining problems are reassigned, our next topic is hydrodynamics: motion of fluids. There is a short theory inset before the new problems, which we can discuss further in class if necessary.

REASSIGNED HOMEWORK

1. A vessel with water slides down an inclined plane with acceleration  $a$ . The plane makes angle  $\alpha$  with the horizon. What angle does the surface of the water in the vessel make with the horizon?

2. A regular tetrahedron floats under the surface of water in such a way that its bottom face is horizontal and at a depth  $h$ . The tetrahedron side has length  $a$ , water density is  $\rho$ . Find the force exerted by the water on any non-horizontal face of the tetrahedron.



- \*3. A thin-walled metal hemisphere with a little hole at the top rests on a table. Hemisphere's edges fit snugly against the table. Water is being poured inside through the hole and when it rises all the way to the hole, it lifts the hemisphere and starts flowing underneath it. Find the mass of the hemisphere if its' internal radius is  $R$  and density of water is  $\rho$ .



A LITTLE BIT OF THEORY

We want to describe how fluids flow, under the ideal fluid approximation. This approximation means negligible viscosity, or, equivalently, that mechanical energy is not dissipated during fluid motion. Furthermore, we assume that fluid flow is effectively one-dimensional: think about water flowing through a pipe. Finally, we restrict ourselves to a steady case, when the fluid motion has begun a sufficiently long time ago and has already stabilized so that no properties of the flow depend on time.

Our strategy is to consider any two particular cross sections of the pipe and use conservation laws to relate different quantities in these cross sections. Suppose in the first cross section fluid velocity is  $v_1$ , elevation above the ground is  $h_1$ , pressure is  $P_1$  and the area of cross section is  $A_1$ . Correspondingly in the second cross section we have  $v_2, h_2, P_2, A_2$ . There are two relevant conservation laws : mass conservation and energy conservation. Mass conservation law implies:

$$v_1 A_1 = v_2 A_2$$

Energy conservation law implies the following equation, called Bernoulli's law:

$$(1) \quad P_1 + \rho g h_1 + \frac{\rho v_1^2}{2} = P_2 + \rho g h_2 + \frac{\rho v_2^2}{2}$$

In particular, Bernoulli's law implies the usual pressure variation with depth  $P(h) = P_0 + \rho g h$  is the case of static fluid (when velocities are 0).

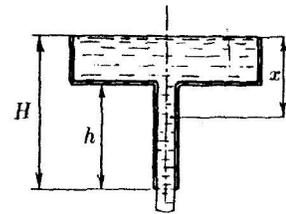
Let us apply Bernoulli's law to find the speed of the liquid flowing out of an opening in an open vessel. Choose one cross section for the Bernoulli's law application to be at the highest point of the liquid and choose the other one to be just outside the opening. Note that at both of these points water is in direct contact with the atmosphere, so its pressure must be equal to the atmospheric pressure. From Bernoulli law we then obtain:

$$(2) \quad v = \sqrt{2gH}$$

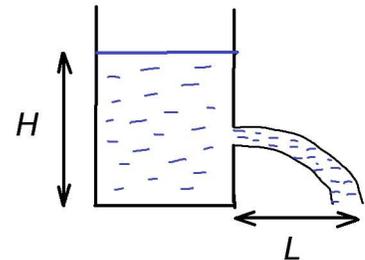
where  $H$  is the height of water level in the vessel as measured from the opening. This is called Torricelli's equation.

#### NEW HOMEWORK

1. Try to derive Bernoulli's law (1) and Torricelli's law (2) from the conservation of mechanical energy of the fluid.
2. Liquid of density  $\rho$  flows out of a wide tank through a narrow tube at the bottom. How do pressure and speed of the fluid depend on the vertical coordinate  $x$ ? Atmospheric pressure is  $P_0$ , dimensions of the tank and the tube are shown on the picture.



3. A wide stream of water flows down a long inclined plane. A group of students measures depth of the stream at some point. Then they measure depth at another point, distance  $l$  down along the plane from the first one, and it turns out to be a half of the depth at the initial point. What distance down the flow does depth become four times smaller?
- \*4. A tank is filled with water up to height  $H$ . We want to drill a hole in a side wall of the tank in such a way that the jet of water produced will land the farthest from the tank. Where should we drill the hole? The tank sits on a horizontal plane.



#### FOR THE NEXT MEETING

**IMPORTANT:** The next club's meeting is at 2:40pm, in person, on Sunday, **March 1**.