Homework 9.

Kirchhoff's current law. Nodal analysis: a way to solve DC circuits.

We will discuss how to solve simple direct current (DC) circuits. "DC" means that all the voltages and current do not change in time. We know three circuit elements: resistor, ideal voltage source and ideal wire. To enforce an electrical current through a resistor, we must apply to the resistor a finite potential difference (voltage). According to Ohm's law, the current I is proportional to the voltage U, and the coefficient of proportionality is the resistance R:

$$U = I \cdot R \qquad (1)$$

A resistor is usually shown as a zigzag line: 

An ideal voltage source maintains a fixed potential difference between its terminals. This means that if, say, a 3-volt ideal voltage source is connected to points A and B, the voltage difference between the points (potential in point A minus potential in point B) will be 3 volts, no matter what else is connected to them.

An ideal voltage source is shown as: 

An ideal voltage source can accommodate any current. A good way to think about the functioning of an ideal voltage source is that it adjusts its current to maintain fixed voltage between its terminals.

An ideal wire connects the circuit elements. It has zero resistance; hence the current can flow through an ideal wire without a voltage drop across it. If two points of a circuit are connected with an ideal wire, the potential difference between them is zero.

An ideal wire is shown as a solid line: 

We also discussed Kirchhoff's voltage law. It says:

Sum of the voltage drops along any closed loop of the circuit has to be zero. You can choose the direction (clockwise or counterclockwise – it is arbitrary, but the result will not depend on your choice). Then, moving along the loop in the direction you have chosen, you sum up all the "voltage drops" along the loop. The sign of the voltage drop or the voltage source could be chosen according to the following way: the voltage is positive if you are moving from "minus" to "plus" – it is like "uphill" motion for the positive charge. If you are moving from "plus" to "minus" is like "downhill" motion, and the voltage is negative. In the resistor, the current always flows from "plus" to "minus".

If we assume that the sum of the voltages around a close loop is nonzero and equals ΔU, then as the charge ***q*** *moves around this loop and returns to the starting point it earns the potential energy P=q ΔU. But the potential energy depends only on the position of the object and does not depend on the way we use to bring the object to this position, so ΔU has to be zero.*

This l;aw is known as ***Kirchhoff current law***. They named after German physicist Gustav Kirchhoff.

Gustav Kirchhoff (1824-1887)

To find the current in a simple circuit consisting from one loop, like one shown in Figure 2, you can do the following steps:

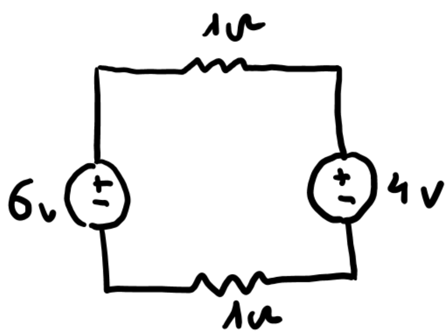
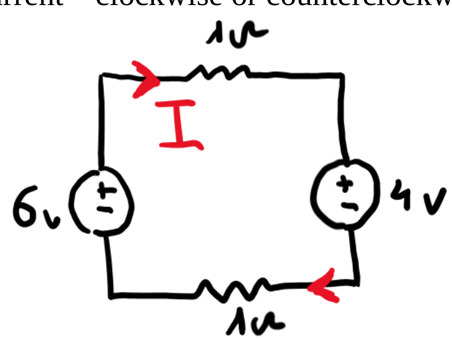


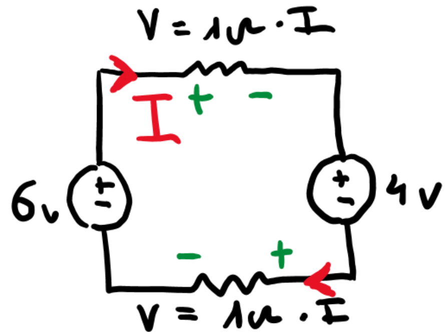
Figure 2.

1. Choose the direction of the current – clockwise or counterclockwise – it is up to you.



2. Mark “plus” and “minus” sides of the resistors according to your choice of the current direction: the current enters into “plus “ and exits from “minus”.

I



3. Sum up the voltages along the loop and make the sum equal to zero. This will give you equation with the current as the variable:

$$6 - 1 \cdot I - 4 - 1 \cdot I = 0$$

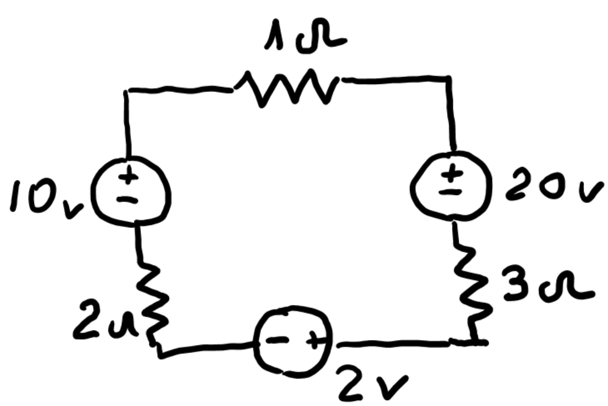
4. Solve the equation and find the current magnitude:

$$6 - 1 \cdot I - 4 - 1 \cdot I = 0$$
$$2 - 2 \cdot I = 0$$
$$I = 1A.$$

If the current is positive, then you guessed the current direction right, if the current is negative, then it flows against your chosen direction.

Problems:

1. Find the current in the circuit shown below. Choose the current direction counter-clockwise.



2. Solve the problem 1 by choosing the current direction clockwise.