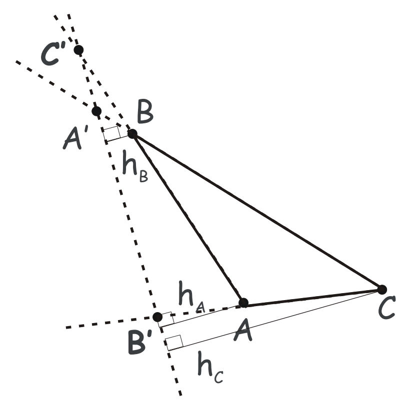
Geometry.

Review the last classwork handout on solving problems using mass points and the center of mass. Solve the unsolved problems from previous homeworks. Try solving the following problems (skip the ones you have already solved).

Problems.

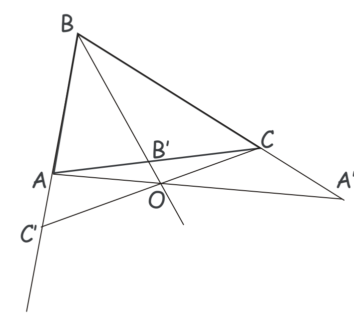
1. Prove Menelaus theorem for the configuration shown on the right using mass points. Menelaus theorem states,



Points C', A' and B', which belong to the lines containing the sides AB, BC and CA, respectively, of triangle ABC are collinear if and only if,

$$\frac{|AC'|}{|C'B|}\frac{|BA'|}{|A'C|}\frac{|CB'|}{|B'A|}=1$$

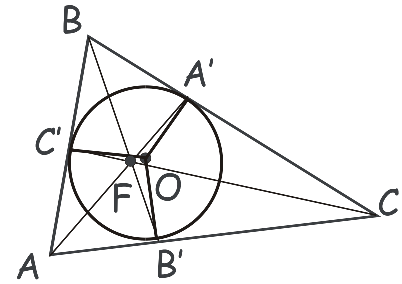
2. Prove the extended Ceva theorem (i) using mass points and the center of mass and (ii) using the similarity of triangles. Extended Ceva theorem states,



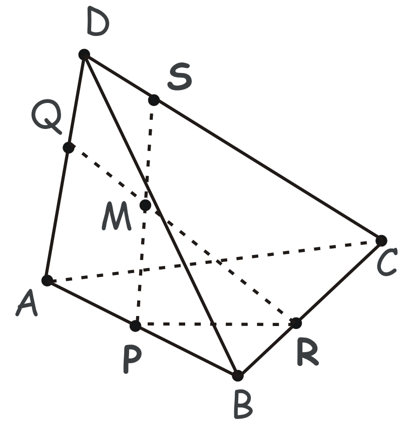
Segments (Cevians) connecting vertices A, B and C, with points A', B' and C' on the sides, or on the lines that suitably extend the sides BC, AC, and AB, of triangle ABC, are concurrent if and only if,

$$\frac{|AC'|}{|C'B|}\frac{|BA'|}{|A'C|}\frac{|CB'|}{|B'A|}=1$$

3. In a triangle ABC, A', B' and C' are the tangent points of the inscribed circle and the sides BC, AC, and AB, respectively (see Figure). Prove that cevians AA', BB' and CC' are concurrent (their common point F is called the Gergonne point).

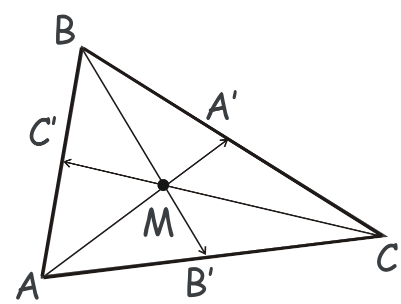


4. Consider segments connecting each vertex of the tetrahedron $ABCD$ with the centroid of the opposite face (the crossing point of its medians). Prove that all four of these segments, as well as the segments connecting the midpoints of the opposite edges (opposite edges have no common points; there are three pairs of opposite edges in a tetrahedron, and therefore three such segments) – seven segments in total, have common crossing point (are concurrent).



5. In a quadrilateral $ABCD$, E and F are the mid-points of its diagonals, while O is the point where the midlines (segments conneting the midpoints of the opposite sides) cross. Prove that E, F, and O are collinear (belong to the same line).

6. In a triangle ABC, Cevian segments AA', BB' and CC' are concurrent and cross at a point M (point C' is on the side AB, point B' is on the side AC, and point A' is on the side BC). Given the ratios $\frac{AC'}{C'B} = p$ and $\frac{AB'}{B'C} = q$, find the ratio $\frac{AM}{MA'}$ (express it through p and q).



7. What is the ratio of the two segments into which a line passing through the vertex A and the middle of the median BB' of the triangle ABC divides the median CC'?

8. In a parallelogram $ABCD$, a line passing through vertex D passes through a point E on the side AB, such that $|AE|$ is $1/n$-th of $|AB|$, n is an integer. At what distance from A, relative to the length, $|AC|$, of the diagonal AC it meets this diagonal?

9. Points P and Q on the lateral sides AB and BC of an isosceles triangle ABC divide these sides into segments whose lengths have ratios $|AP|:|PB| = n$, and $|BQ|:|QC| = m$. Segment PQ crosses altitude BB' at point M. What is the ratio $|BM|:|MB'|$ of two segments into which PQ divides the altitude BB'?