

MATH 7: HANDOUT 5
ARITHMETIC SEQUENCES

A sequence of numbers is an **arithmetic sequence** or **arithmetic progression** if the difference between consecutive terms is the same number, the **common difference** or d .

Example: The sequence 1, 5, 9, 13, 17, ... is an arithmetic sequence because the difference between consecutive terms is $d = 4$.

We can also find the n -th term if we know the 1st term and d ?

Example: What is a_{100} in the example above?

$$a_1 = 1$$

$$a_2 = a_1 + d = 1 + 4 = 5$$

$$a_3 = a_2 + d = (a_1 + d) + d = a_1 + 2d = (1 + 4) + 4 = 1 + 2 \times 4 = 9$$

$$a_4 = a_3 + d = (a_2 + d) + d = ((a_1 + d) + d) + d = a_1 + 3d = 1 + 3 \times 4 = 13$$

The pattern is:

$$a_n = a_1 + (n - 1) \cdot d$$

$$a_{100} = a_1 + 99d = 1 + 99 \times 4 = 397$$

Properties of an Arithmetic Sequence. A useful property of an arithmetic sequence is that any term is the arithmetic mean of its neighbors:

$$a_n = \frac{a_{n-1} + a_{n+1}}{2}$$

Proof:

$$a_n = a_{n-1} + d$$

$$a_n = a_{n+1} - d$$

Adding these two equalities gives us:

$$2a_n = a_{n-1} + a_{n+1}$$

from where we can get what we need.

Another property of arithmetic sequences is that we can find the common difference d if we know any two terms a_m and a_n :

$$d = \frac{a_m - a_n}{m - n}$$

HOMEWORK

1. Write the first 5 terms of an arithmetic sequence if $a_1 = 7$ and $d = 2$.
2. What are the first 2 terms for the sequence

$$a_1, a_2, -9, -2, 5, \dots?$$

3. $a_{10} = 131$ and $d = 12$. What is a_1 ?
4. $a_5 = 27$ and $a_{27} = 60$. Find the first term a_1 and the common difference d .
5. Find the common difference d in an arithmetic sequence if the 9-th term is 18 and the 11-th term is 44.
6. In the arithmetic progression 5, 17, 29, 41, ... what term has a value of 497?
7. In a given arithmetic progression, the first term is 6, and the 87-th term is 178. Find the common difference of this arithmetic progression, and give the value of the first five terms.
8. The 3-rd term of the arithmetic progression is equal to 1. The 10-th term of it is three times as much as the 6-th term. Find the first term and the common difference. (**Hint:** Use the formula for the n -th term of the progression and write what is given in the problem using this formula.)