MATH 7: HOMEWORK 18
Coordinate geometry review.
March 16, 2025

1. Coordinate geometry: Introduction.

In this section of the course, we are going to study coordinate geometry. The basic notion is the **coordinate plane**– a plane with a given fixed point, called the **origin**, as well as two perpendicular lines – **axes**, called the x-**axis** and the y-**axis**. x-axis is usually drawn horizontally, and y-axis — vertically. These two axes have a **scale** – "distance" from the origin.

The scales on the axes allow us to describe any point on the plane by its **coordinates**. To find coordinates of a point P, draw lines through P perpendicular to the x- and y-axes. These lines intersect the axes in points with coordinates x_0 and y_0. Then the point P has x-coordinate x_0, and y-coordinate y_0, and the notation for that is: $P(x_0, y_0)$.

The **midpoint** M of a segment AB with endpoints A (x_1, y_1) and B (x_2, y_2) has coordinates: $M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

2. Lines

Given some relation which involves variables x, y (such as $x + 2y = 0$ or $y = x^2 + 1$), we can plot on the coordinate plane all points $M(x, y)$ whose coordinates satisfy this equation. Of course, there will be infinitely many such points; however, they usually fill some smooth line or curve. This curve is called the **graph** of the given relation.

Every relation (**equation**) of the form: $y = \boldsymbol{mx + b}$

where $\boldsymbol{m}$, $\boldsymbol{b}$ are some numbers, defines a straight line. The slope of this line is determined by m: as you move along the line, y changes m times as fast as x, so if you increase x by 1, then y will increase by m. In other words, given two points $A(x_1, y_1)$ and $B(x_2, y_2)$ **slope** can be computed by dividing change of y: $\Delta = y_2 - y_1$ by the change of x: $\Delta = x_2 - x_1$:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

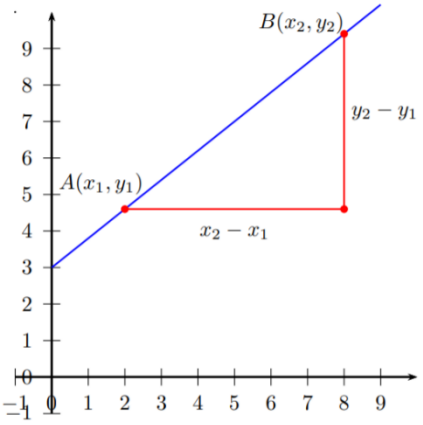


Fig. 1

Two non-vertical lines are **parallel** if and only if they have the **same slope**.

In the equation $y = mx + b$, b is a y-**intercept**, and determines where the line intersects the vertical axis (y-axis). The equation of the **vertical** line is $x = k$, and the equation of the **horizontal** line is $y = k$. Notice that in case of the vertical line, the slope is undefined.

3. Distance between points. Circles.

The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by the following formula:

$$\boldsymbol{d = \sqrt{(x_2 - x_2)^2 + (y_2 - y_1)^2}}$$

This formula is a straightforward consequence of the Pythagoras' Theorem (Fig. 1).

The equation of the circle with the center $M(x_0, y_0)$ and radius $\boldsymbol{r}$ is:

$$(x - x_0)^2 + (y - y_0)^2 = r^2.$$

This equation means, that points (x, y) should be at distance $\boldsymbol{r}$ from the given point $M(x_0, y_0)$.

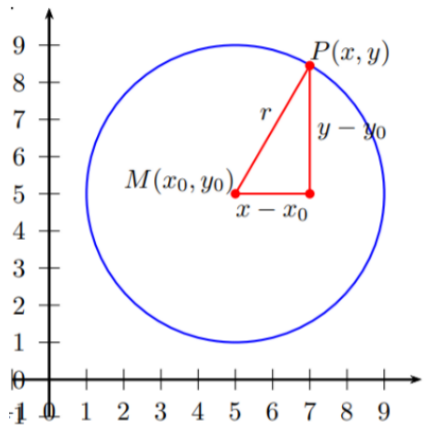


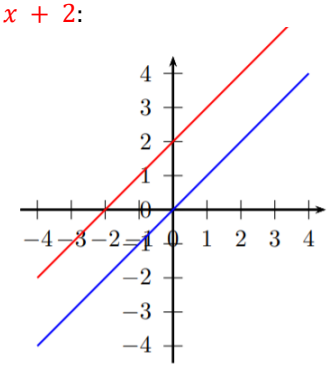
Fig.2

4. Graphs of functions

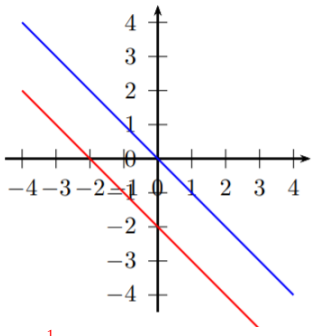
In general, the relation between x and y could be more complicated and could be given by some formula of the form $y = f(x)$, where f is some function of x (i.e., some formula which contains x). Then the set of all points whose coordinates satisfy this relation is called the **graph** of f.

Line. The graph of the function $y = mx + b$ is a straight line. The coefficient ***m*** is called the *slope*.

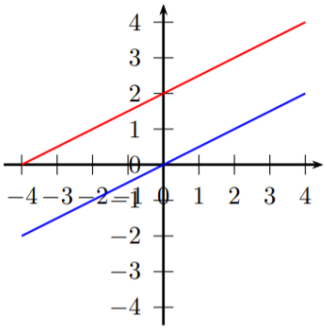
$y = x$; $y = x + 2$:



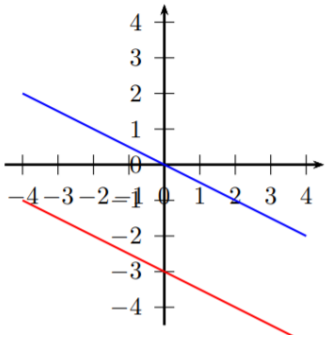
$y = -x$; $y = -x - 2$:



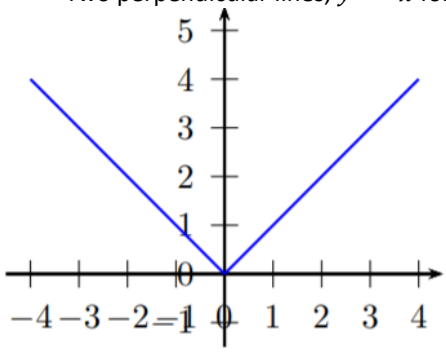
$y = \frac{1}{2}x$; $y = \frac{1}{2}x + 2$:



$y = -\frac{1}{2}x$; $y = -\frac{1}{2}x - 3$:



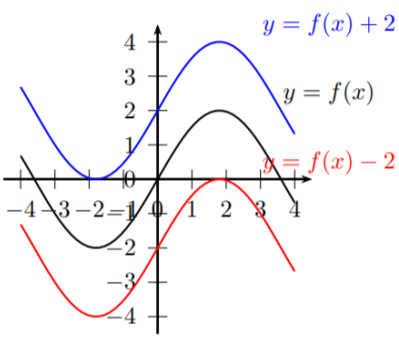
Absolute value of a line. $y = |x|$ Two perpendicular lines, $y = x$ for $x > 0$ and $y = -x$ for $x < 0$.

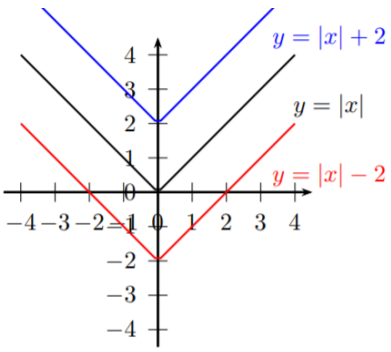


5. Function transformations

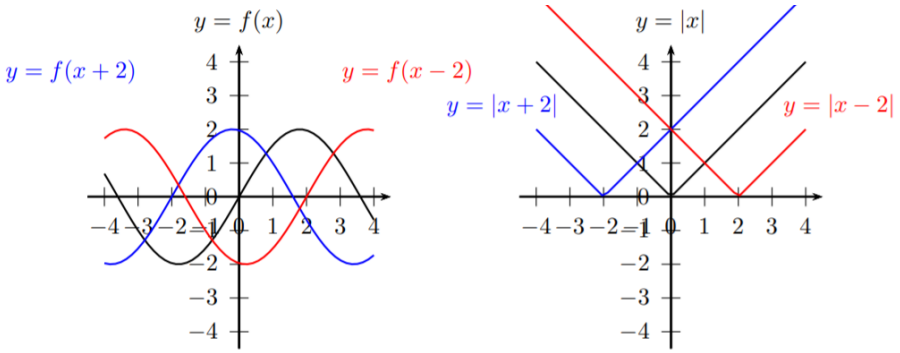
Having learned a number of basic graphs, we can produce new graphs, by doing certain transformations of the equations. Here are two of them.

Vertical translations: Adding constant ***c*** to the right-hand side of equation shifts the graph by ***c*** units up (if c is positive; if c is negative, it shifts by |c| down.)





Horizontal translations: Adding constant ***c*** to x shifts the graph by ***c*** units left if ***c*** is positive; if ***c*** is negative, it shifts by ***c*** right.



Homework problems

ALL GRAPHS/POINTS/FIGURES SHOULD BE DRAWN BY YOU - NOT PRINTED! USE QUADRILE PAPER!

Note: to graph – means use a few pairs of points to graph the line/curve of the function.
to sketch – produce a sketch which **approximates** main features of the function, using the equation of the function and other properties (e.g., use **slope, intercept, shifts, vertex place …)**

1. A point B is 5 units above and 2 units to the left of point $A(7,5)$. What are the coordinates of point B?

2. Find the coordinates of the midpoint of the segment AB, where $A = (3,11), B = (7,5)$.

3. Draw points $A(4,1)$, $B(3,5)$, $C(-1,4)$. If you did everything correctly, you would get 3 vertices of a square. What are the coordinates of the fourth vertex? What is the area of this square?

4. 3 points $(0,0)$, $(1,3)$, $(5,-2)$ are the three vertices of a parallelogram. What are the coordinates of the remaining vertex? (Hints: check the slopes of each line.)

5. Consider the triangle ΔABC with the vertices $A(-2,-1)$, $B(2,0)$, $C(2,1)$. Find the coordinates of the midpoint of B and C. Find the length of the median (i.e., a median unites a vertex with the midpoint of the opposite side) from A in the triangle ΔABC.

6. What is the slope of a line whose equation is $y = 2x$? What is the slope of the line $y = mx$?

7. In this problem you will find equations that describe some lines.
   a. What is the equation whose graph is the $y-axis$?
   b. What is the equation of a line whose points all lie 5 units above the $x-axis$?
   c. Is the graph of $y = x$ a line? Draw it.
   d. Find the equation of a line that contains the points $(1,-1)$, $(2,-2)$, and $(3,-3)$.

8. For each of the equations below, draw the graph, then draw the perpendicular line (going through the point (0, 0)) and then write the equation of the perpendicular line

   a. $y = 3x$ b. $y = -x$ c. $y = -\frac{1}{2}x$

Can you determine the general rule: if the slope of a line is k, what is the slope of the perpendicular line?

9. Find the equation of the line through $(1, 1)$ with slope 2.
10. Find the equation of the line through points $(1, 1)$ and $(3, 7)$. [Hint: what is the slope?]

11. (a) Find k if $(1, 9)$ is on the graph of $y - 2x = k$. Sketch the graph.
    (b) Find k if $(1, k)$ is on the graph of $5x + 4y - 1 = 0$. Sketch the graph.
    A line written in this form, $Ax + By + C = 0$, is known as a *standard form*. When this form could be more useful than the slope – intercept form? Watch here.

12. Let $l1$ be the graph of $y = x + 1$, $l2$ be the graph of $y = x - 1$, $m1$ be the graph of $y = -x + 1$, and $m2$ be the graph of $y = -x - 1$. Graph them.
    a. Find the intersection point of $l1$ and $m1$; Label this point A and write down its coordinates.
    b. Find the intersection point of $l2$ and $m2$; Label this point B and write down its coordinates.
    c. Find the midpoint of AB and write down its coordinates.
    d. Let C be the intersection point of $l1$ with $m2$, and D be the intersection point of $l2$ with $m1$. What kind of quadrilateral is $ABCD$?
    e. Explain why $l1$ and $l2$ are parallel. What is the distance between them?

13. Find the intersection point of a line $y = x - 3$ and a line $y = -2x + 6$ algebraically solving system equations. Then, sketch the graphs of these lines – did the coordinates of the intersecting point match your solution for x and y?

14. Using the shape of the function y = |x|, sketch (do not graph) on paper the graphs of the following functions:
    a. $y = |x| + 1$
    b. $y = |x + 1|$
    c. $y = |x - 5| - 3$