

Math 6: Homework 2.1

Arithmetic Sequences

Arithmetic sequence

A sequence of numbers (typically but not always integers) is an arithmetic sequence if the difference between consecutive terms is the same number, the **common difference**, let's call it d .

For example, let's consider the sequence: 1, 5, 9, 13, 17, ...

The first term in the sequence is $a_1 = 1$, the second is $a_2 = 5$, and so on. The difference is $d = 4$.

What is the n^{th} term? For example what is a_{100} ?

$$a_1 = 1$$

$$a_2 = a_1 + d = 1 + 4 = 5$$

$$a_3 = a_2 + d = (a_1 + d) + d = a_1 + 2d = (1 + 4) + 4 = 1 + 2 \times 4 = 9$$

$$a_4 = a_3 + d = (a_2 + d) + d = ((a_1 + d) + d) + d = a_1 + 3d = 1 + 3 \times 4 = 13$$

....

$$a_n = a_1 + (n - 1)d$$

$$\text{So } a_{100} = a_1 + 99d = 1 + 99 \times 4 = 397$$

Property of an arithmetic sequence

A property of an arithmetic sequence is that any term is the arithmetic mean of its neighbors.

$$a_n = \frac{a_{n-1} + a_{n+1}}{2}$$

Sum of an arithmetic sequence

$$S = a_1 + a_2 + a_3 + \dots + a_n = n \cdot \frac{a_1 + a_n}{2}$$

Problems

1. What are the first 2 terms of the arithmetic sequence $a_1, a_2, -9, -2, 5, \dots$?
2. Find the common difference d in an arithmetic sequence if the 9-th term is 18 and the 11-th term is 44.
3. Find the sum of the first 100 terms if $a_1 = -1$ and $d = 1$.
4. Find the sum of the first 1000 odd numbers.
5. Simplify the following expression:

$$\frac{2}{\frac{1}{1 - \frac{1}{3}} - 1} \div \frac{\frac{1}{2}}{\frac{2}{3} - \frac{1}{4}}$$

6. The 3-rd term of the arithmetic progression is equal to 1. The 10-th term of it is three times as much as the 6-th term. Find the first term and the common difference. (**Hint:** Use the formula for the n -th term of the progression and write what is given in the problem using this formula.)
7. *The sum of the first 20 terms of an arithmetic progression is 200, and the sum of the next 20 terms is -200. Find the sum of the first hundred terms of the progression.