COORDINATE GEOMETRY : FUNCTIONS AND TRANSFORMATIONS

FUNCTIONS

A function is a mathematical construct that takes an input and gives a unique value as an output. For example, consider the following function:

$$f(x) = 2x + 1$$

This function f can take any number, and it will give us an output based on its definition. For example, if we input 2 to our function we would get $f(2) = 2 \times 2 + 1 = 5$. We can repeat this for many numbers:

$$f(0) = 2 \times 0 + 1 = 1, \quad f(3.5) = 2 \times 3.5 + 1 = 8, \quad \text{etc...}$$

A function may be much more complex and it can have many rules as long as it gives us a single result for each input that we feed it with.

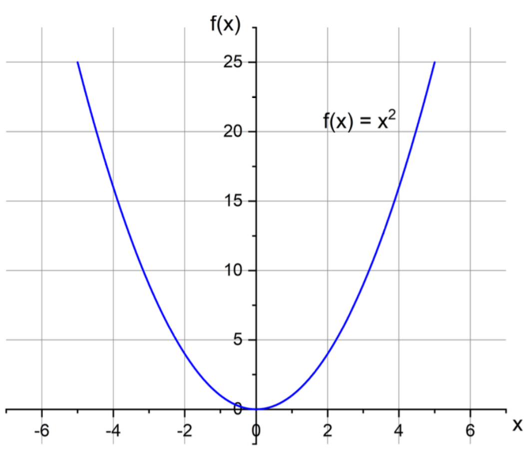
Graph of a function: A great way to understand the behavior of a function is by studying its graph. To do this, we will use the coordinate geometry that we had learned previously. If we decide to write

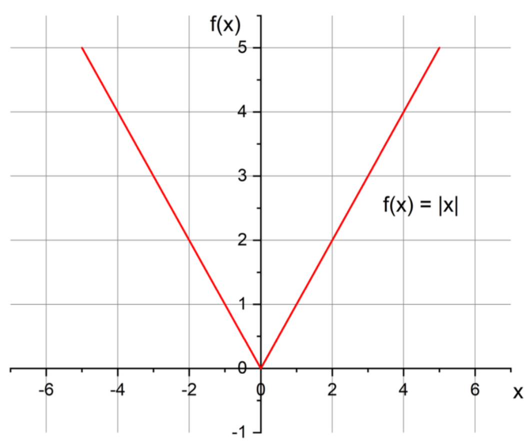
$$y = f(x),$$

then we can make a graph of this function in the same way as we made graphs for other objects in the previous classes. For example, the function which we defined earlier, $f(x) = 2x + 1$, would now be written as

$$y = 2x + 1,$$

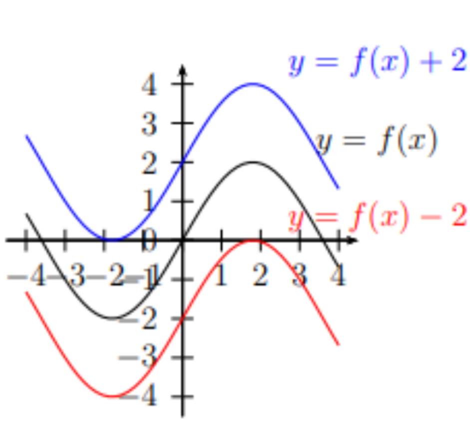
which we know corresponds to the equation of a line. Other interesting functions with nice graphs are $f(x) = x^2$, which is a parabola, and $f(x) = |x|$.

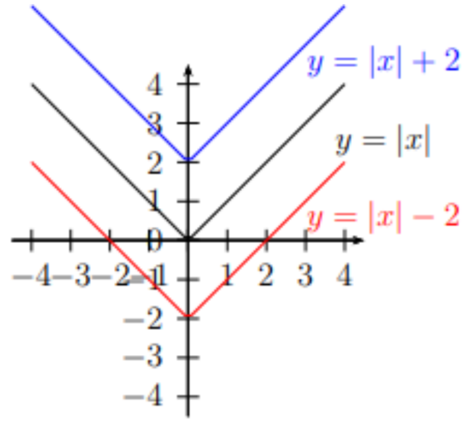




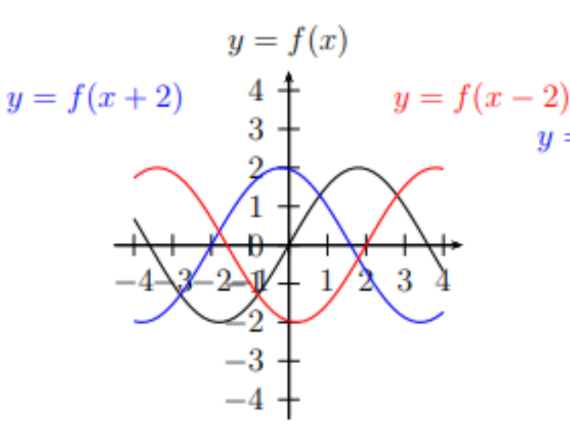
Transformations: Having these basic graphs, we can produce new graphs, by doing certain transformations of the equations. Here are two of them.

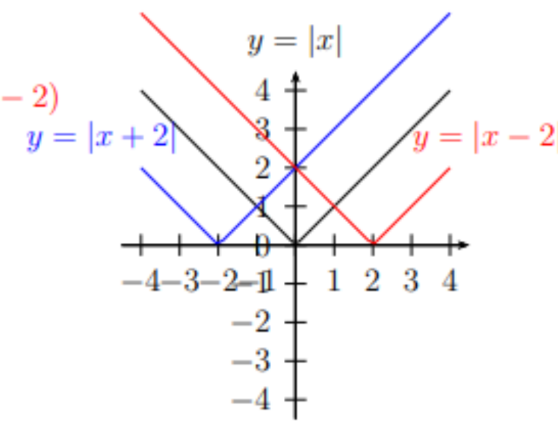
- **Vertical Translations:** Adding constant c to the right-hand side of equation shifts the graph by c units up (if c is positive; if c is negative, it shifts by $|c|$ down.)





- **Horizontal Translations:** Adding constant c to x shifts the graph by c units left if c is positive; if c is negative, it shifts by c right.





Similarly, multiplying x by negative sign i.e. $f(-x)$ is equivalent to mirror reflection about y-axis line. And multiplying the function by $-$ sign i.e. $-f(x)$ is equivalent to mirror reflection about x-axis line.

HOMEWORK

1. (a) Sketch the graphs of functions $y = |x+1|$ and $y = -x + 0.25$ in the same coordinate plane.

(b) How many solutions for x does the following equation have:

$$|x+1| = -x + 0.25$$

Note: you do not have to find the solutions, you just need to know how many solutions it will have.

2. Graph a sketch of the following functions. Remember, you can always make a table of x and y values if you are confused how the function looks like.

(a) $y = |x| + 1$
(b) $y = |x + 1|$
(c) $y = |x - 5| + 1$
(d) $y = -|x + 1|$
(e) $y = |-x + 1|$

3. Graph the function $f(x) = x^3 + x^2 - 2x$ on a graph that goes from -3 to 3. Hint: First, tabulate the corresponding value of $f(x)$ every 0.5 steps and graph these points. Then, try to connect them continuously.

4. Sketch the following function:

$$f(x) = \begin{cases} x^2 & \text{if} \quad x \le 0 \\ x & \text{if} \quad x > 0 \end{cases}$$

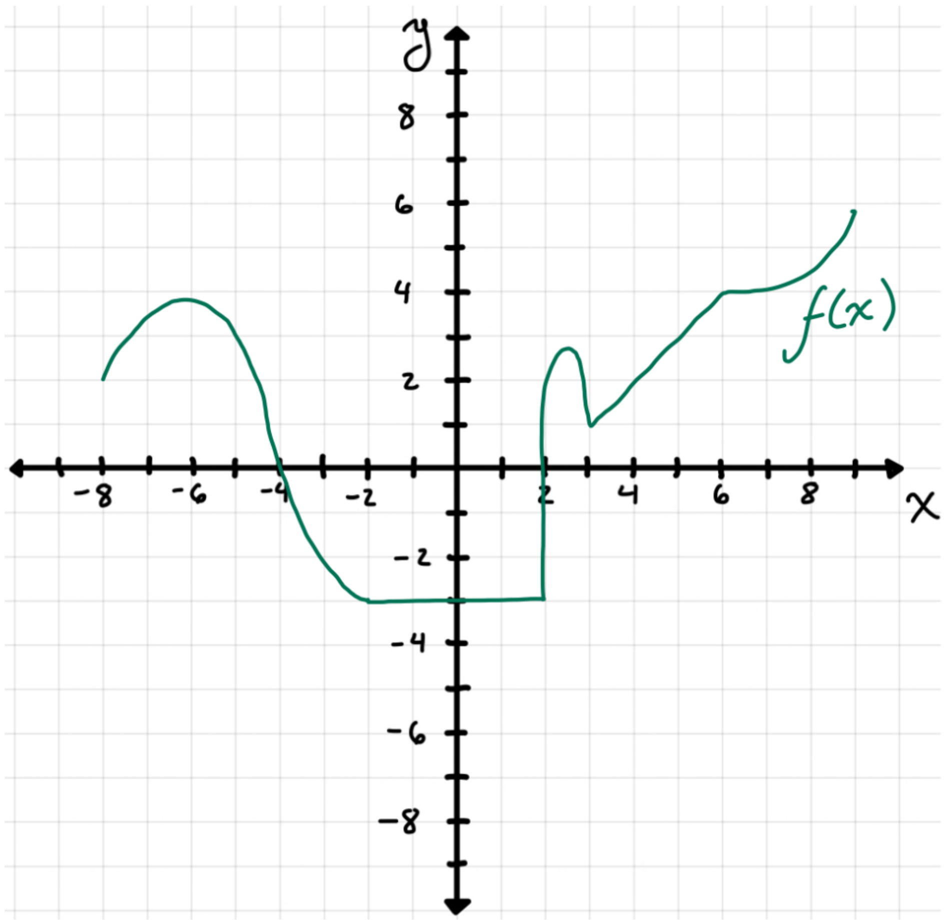


FIGURE 1. Function $f(x)$ for problem 5.

5. Fig. 1 plane shows the graph of a function $f(x)$. Draw the graph of the function $g(x) = f(x) + 2$ on the same coordinate plane. **Note:** you do not need to know how function f is defined.

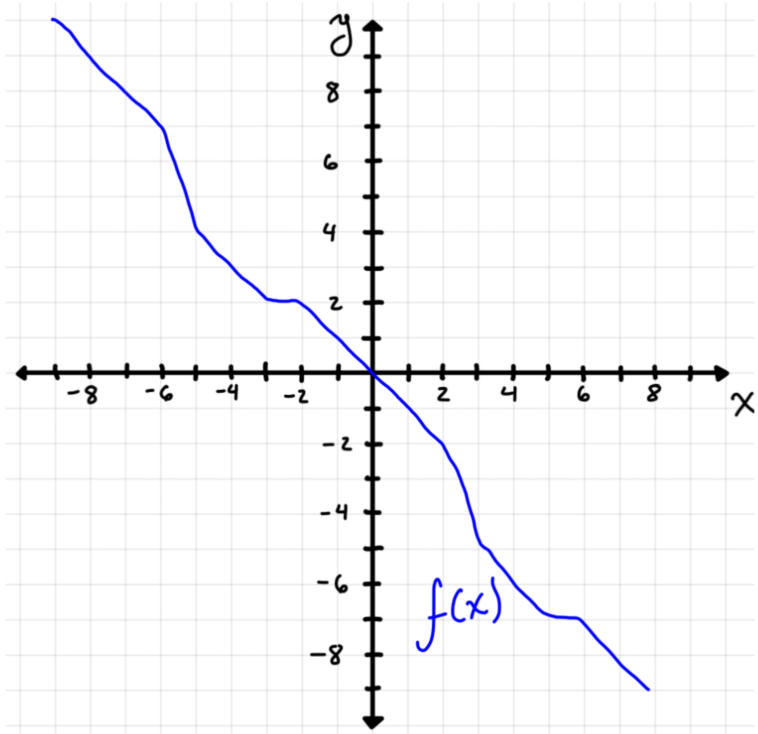


FIGURE 2. Function $f(x)$ for problem 6.

6. Fig. 2 shows the graph of a function $f(x)$. Draw the graph of function $g(x) = f(x - 2)$ on the same coordinate plane. **Note:** you do not need to know how function f is defined.

***7.** One of the most important functions in trigonometry is the $\sin(x)$ function. Later on, you will learn how it is defined and how to use it. For now, use a calculator to tabulate some values of the function and try to sketch it from -10 to 10. How many times does it intersect the x axis in this range?

***8.** Sketch the following functions:
(a) $y = |x| + |x + 1|$
(b) $y = |x - 1| + |x + 1|$
Hint: First, draw the graph for each of the terms being added. Then, try to add the graphs.