MATH 6
HOMEWORK 10: SETS

LOGIC AND COMPUTER CHIPS

I briefly mentioned in the class that logic gates have applications to build very complicated circuits inside computer chips. All the inputs given to the computer can be categorized as 0 (False) or 1 (True) which means either you have a zero or non-zero voltage across different components (transitors) inside your computer chips. The usual convention is

- Positive voltage=true
- Zero voltage=false

Then one can relatively easily construct AND, NOT,... chips, and combining them, more complicated chips. If you are interested to learn more about how logic gates are used to build circuits inside computers, you can watch the following video on youtube: *https://www.youtube.com/watch?v=Sc3lh3D4rCw*

SETS

Describing Sets. By word *set*, we mean any collection of objects: numbers, letters, ... Most of the sets we will consider will consist either of numbers or points in the plane. Objects of the set are usually referred to as *elements* of this set.

Sets are usually described in one of two ways:

- By explicitly listing all elements of the set. In this case, curly brackets are used, e.g. $\{1, 2, 3\}$.
- By giving some conditions, e.g. "set of all numbers satisfying equation $x^2 > 2$". In this case, the following notation is used: $\{x \mid \dots\}$, where dots stand for some condition (equation, inequality, ...) involving x, denotes the set of all x satisfying this condition. For example, $\{x \mid x^2 > 2\}$ means "set of all x such that $x^2 > 2$".

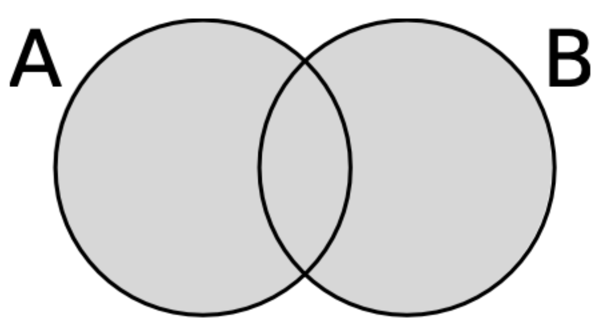
Members of sets. Sometimes we might have to say whether the element belongs to the set or not. In this case the following notation is used:

- $x \in A$ means "x is in A", or "x is an element of A"
- $x \notin A$ means "x is not in A"

Set Operations. There are several operations that can be used to get new sets out of the old ones:

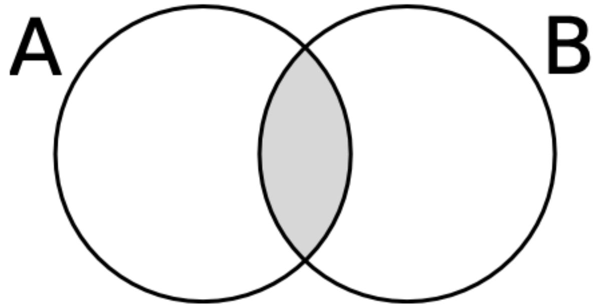
- $A \cup B$: *union* of A and B. It consists of all elements which are in either A or B (or both):

$$A \cup B = \{x \mid x \in A \text{ OR } x \in B\}.$$



- $A \cap B$: *intersection* of A and B. It consists of all elements which are in both A and B:

$$A \cap B = \{x \mid x \in A \text{ AND } x \in B\}.$$

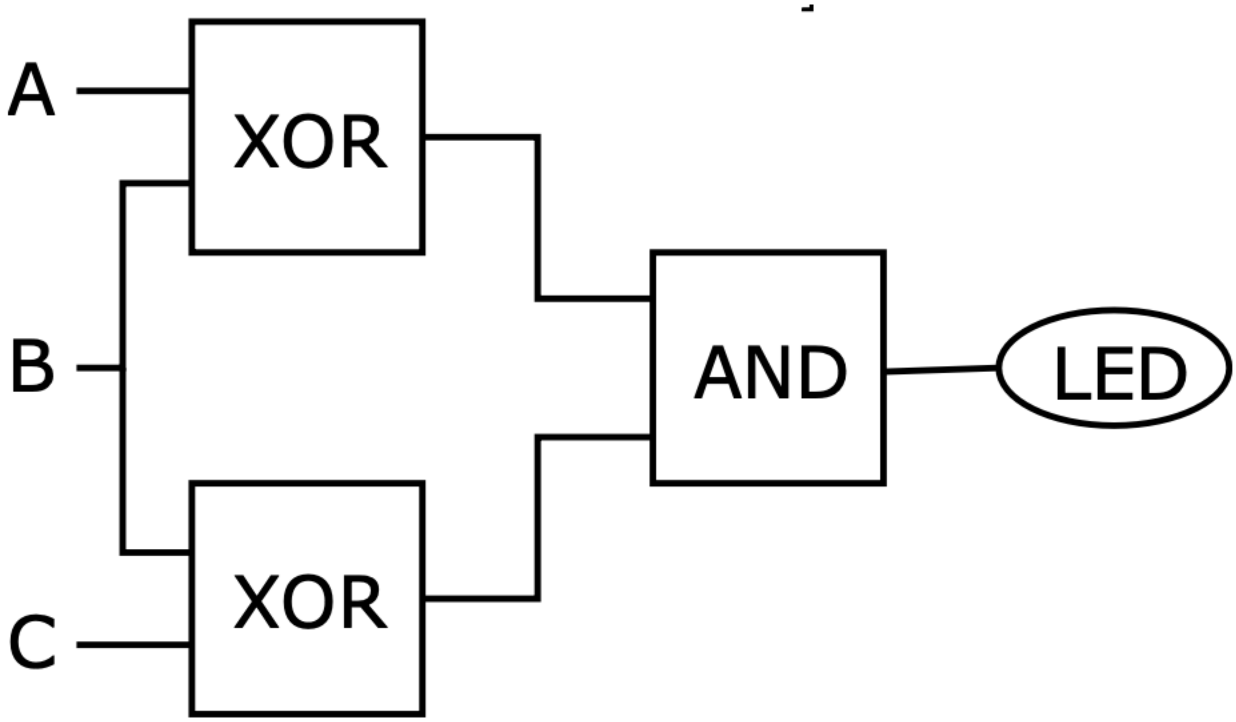


- $\overline{A}$: *complement* of A, i.e. the set of all elements which are not in A: $\overline{A} = \{x \mid x \notin A\}$.

Intervals. The following notations are used when we talk about intervals on the number line. Intervals may have end points included or excluded: [and] represent that the end point is included, while (and) indicate that the end point is excluded.

$[a, b] = \{x \mid a \leq x \leq b\}$ is the interval from a to b (including endpoints),
$(a, b) = \{x \mid a < x < b\}$ is the interval from a to b (**not** including endpoints),
$[a, \infty) = \{x \mid a \leq x\}$ is the half-line from a to infinity (including a),
$(a, \infty) = \{x \mid a < x\}$ is the half-line from a to infinity (**not** including a)

1. The diagram below shows some circuit constructed of 3 logical chips (each with two inputs and one output; we draw them so that the inputs are on the left and the output, on the right). Can you determine for which values of inputs the LED will light up? [Hint: this is the same a writing a truth table for some formula....]



Note: the wires connecting each of the chips and LED to the power source are not shown.

2. Consider the operation NOR which is just the opposite of OR: it returns T only if both A and B are F. Using only the component NOR, see if you can create circuits equivalent to AND, OR, and NOT as you did in the previous problem.

3. Using only AND, NOT, and OR, produce a three-input AND circuit, i.e., the output is F unless all three inputs are a T. (You do not have to use all three circuit elements.)

4. If Al comes to a party, Betsy will not come. Al never comes to a party where Charley comes. And either Betsy or Charley (or both) will certainly come to the party.

Based on all of this, can you explain why it is impossible that Al comes to the party?

5. Let

- A=set of all people who know French
- B=set of all people who know German
- C=set of all people who know Russian

Describe in words the following sets:

(a) $A \cap B$ (b)$A \cup (B \cap C)$ (c) $(A \cap B) \cup (A \cap C)$ (d) $C \cap \overline{A}$.

6. Let us take the usual deck of cards. As you know, there are 4 suits, hearts, diamonds, spades and clubs, 13 cards in each suit.

Denote:

- H=set of all hearts cards
- Q=set of all queens
- R=set of all red cards

Describe by formulas (such as $H \cap Q$) the following sets:

- all red queens
- all black cards
- all cards that are either hearts or a queen
- all cards other than red queens

How many cards are there in each set?

7. In a class of 25 students, 10 students know French, 5 students know Russian, and 12 know neither. How many students know both Russian and French?

8. Draw the following sets on the number line:

(a) Set of all numbers x satisfying $x \leq 2$ and $x \geq -5$;

(b) Set of all numbers x satisfying $x \leq 2$ or $x \geq -5$

(c) Set of all numbers x satisfying $x \leq -5$ or $x \geq 2$

9. For each of the sets below, draw it on the number line and then describe its complement:

(a) $[0, 2]$ (b) $(-\infty, 1] \cup [3, \infty)$ (c) $(0, 5) \cup (2, \infty)$