Potential exam problems
May 11, 2025

1. Factorize
   a. $(x-3)^2-(y+3)^2$
   b. $m^2 - m - 12$
   c. $a^4 - 144b^{10}$
   d. x^4+4
   e. $22x^2 + 47x + 21$
   f. p^4+4z^{4n}

2. Solve for x:
   a. $x^2 + 5x = 11/4$
   b. $4m^2 - 49p^2q^2 = 0$
3. The 3-rd term of the arithmetic progression is equal to 1. The 10-th term of it is three times as much as the 6-th term. Find the first term and the common difference.
4. Write the first 5 terms of a geometric progression if $a_1 = -25$ and $q = \frac{1}{3}$
5. What is the common ratio of the geometric progression: $\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \ldots$? What is a_{10}? What is a_{100}?
6. Calculate the sum of infinite series: $\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\cdots$
7. What is the sum of: $1 + x + x^2 + x^3 + x^4 + x^5 + \cdots + x^{100}$?
8. A geometric progression has 99 terms, the first term is 12 and the last term is 48. What is the 50-th term?
9. Solve the following equations. Remember that this equation will have two solutions due to the absolute value. (a) $|x-8|=12$ (b) $|6x-1|=3$
10. Write equation of a line passing through point (4,4) and parallel to the line $y = 7/4x - 4$. What is the equation for perpendicular line.
11. Find the equation of the line through $(1,1)$ with slope 2.
12. Find the intersection point of a line $y = x - 3$ and a line $y = -2x + 6$. Sketch the graphs of these lines.
13. Show that opposite angles of a parallelogram are equal.
14. In the country of RGB, there are 13 red, 15 green and 17 blue chameleons. Whenever two chameleons of different colors meet, both of them change their color to the 3rd one (e.g., if red and green meet, they both turn blue). Do you think it can happen that after some time, all chameleons become the same color?[Hint: give each color a numeric value, say 0, 1, 2]

15. There are 16 glasses on a table, arranged in a 4x4 grid, the glass in the bottom-left corner upside down. You are allowed to turn over any 2x2 square of glasses at a time. Can you get all the glasses standing correctly except the one in the top-right corner?

16. Construct a rectangle with one side a and diagonal d.

17. Given a triangle $\triangle ABC$, construct a circle inscribed in the triangle (*Hint:* The center of the circle will be at the intersection of the angle bisectors of the triangle).

18. Construct a right triangle, given a hypotenuse h and one of the legs a

19. Let A be the intersection point of lines l, m, and let angles 1, 3 are vertically opposite angles. Then show that $m\angle 1 = m\angle 3$.

20. Consider three lines l_1, l_2 and m. If l_1, l_2 be perpendicular to m, then $l_1 \parallel l_2$. Conversely,if $l_1 \perp m$ and $l_2 \parallel l_1$, then prove that $l_2 \perp m$.