MATH 6: HANDOUT 19
ODD AND EVEN

CLASSWORK

1. Each Martian has three hands. Can 21 Martians hold their hands (one hand holds only one other hand) so that none of the hands is free?
Solution: Total number of hands is 21, and if these Martians hold hands, then the hands should be broken into pairs, which is impossible.

2. The difference of two integer numbers was multiplied by their product. Is it possible to get 2019?
Solution: $ab(a - b)$ is always even: if either a or b is even, it is obvious; if both a and b are odd, their difference is even.

3. There are 21 coins on the table, all head up. Each turn you're allowed to turn any 20 coins. Is it possible to end up with all coins being tails us?
Solution: To get all tails, each coin needs to be turned odd number fo times, so the total number fo turns needs to be odd as well ($21 \times$ odd number = odd number). But we do even number of turns each time, so the total number of turns would be even. Contradiction.

4. Find two prime numbers so that their sum and their difference is also prime.
Solution: Both prime numbers a and b can't be odd, because then their sum won't be prime. So one of them is even, i.e. $a = 2$. Therefore, if the other number b is prime, then $b - 2, b, b + 2$ are all prime. Among three numbers like that at least one is divisible by 3. The only option is $3, 5, 7$, so the original numbers are 2 and 5.

HOMEWORK

1. In a certain country the parliament has 400 members. After voting for a certain bill, the chairman announced that the bill was approved: there were 27 more votes in favor of the bill than against. The opposition claims that the voting results were falsified. Why? [All 400 parliament members voted, and there were no abstainers.]

2. A bag of 300 hundred gold coins has only coins with values of 1, 3, 5 and 15 piasters. The note on the bag says that the total is 1001 piaster. Can it be correct?

3. The numbers 1 through 10 are written on the blackboard in a row, with spaces left between them. Adam and Bill are playing the following game: on his turn, each player puts either + or - between two numbers. After all signs are written (so they get something like $1 + 23 + 4 + 56\ldots$), the total is computed. If it is even, Adam wins; if it odd, Bill. What is the best strategy for Adam? should he take the first turn or leave the first turn to Bill?

4. A grasshopper is jumping along the number line: the first jump is 1 cm long, the second one, 2 cm, and so on. Can he return to his starting position after 9 jumps? 10 jumps? 2021 jumps?

5. The numbers 1 through 6 are written on the board. You can add 1 to two of the numbers. By repeating this many times, can you make all numbers equal? [**Hint:** what is the sum of the numbers? how does it change?]

6. Can you connect 2021 computers with cables so that each computer is connected to exactly 3 other ones?[**Hint:** how many cables you would need?]

7. A train consists of a locomotive and five cars marked I, II, III, IV and V. In how many ways can you rearrange the cars, in such a way that car I is always closer to the locomotive than car II?