MATH 6: HANDOUT 23
FACTORIZATION. DIVISION OF EXPRESSIONS.

Factorization

When handling with large algebraic expressions, it is often possible to simplify them. One of of doing this is by **factorization**. As its name suggests, this method consists of finding a common factor in two or more terms. For example, in the following expression

$$7x + 9x - 5x$$

the three terms share the common factor x. Therefore, we can rewrite this expression as:

$$7x + 9x - 5x = (7 + 9 - 5)x = 11x.$$

In general, we will have the following identities:

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a+b)(a-b) = a^2 - b^2$$

$$ab + ac = a(b+c)$$

Polynomial division

In arithmetics, we divide numbers using long division. We can do the similar process for dividing algebraic expressions.

For example, we can divide $2x^2 - 7x + 3$ by $x - 3$

$$\begin{array}{r|l} & 2x - 1 \\ \hline x - 3 & 2x^2 - 7x + 3 \\ & \underline{-\,2x^2 + 6x} \\ & -\,x + 3 \\ & \underline{x - 3} \\ & 0 \end{array}$$

This means that

$$2x^2 - 7x + 3 = (x-3)(2x-1)$$

Now, if we want to solve an equation

$$2x^2 - 7x + 3 = 0$$

we can equate both factors to 0, and get two solutions: when $x - 3 = 0$, then $x = 3$, and when $2x - 1 = 0$, then $x = 1/2$.

This can be useful if we want to solve an equation for which we don't have any good method. For example, if we are trying to solve an equation

$$2x^2 + 7x - 22 = 0$$

we can try to guess one of the solution using guess-and-check method. When we check small whole numbers, we can see that $x = 2$ is a solution, and that would mean that we can divide $2x^2 + 7x - 22$ by $x - 2$:

$$\begin{array}{r|l} & 2x+11 \\ \hline x-2 & 2x^2 + 7x - 22 \\ & -2x^2 + 4x \\ \hline & 11x - 22 \\ & -11x + 22 \\ \hline & 0 \end{array}$$

This means that

$$2x^2 + 7x - 22 = (x-2)(2x+11)$$

and to solve this equation we have to equate two factors to 0: $x - 2 = 0$ gives us $x = 2$, and $2x + 11 = 0$ gives us $x = -5.5$.

Note that as with division of numbers, in some cases you may end up with a remainder:

$$\begin{array}{r|l} & 2x+15 \\ \hline x-3 & 2x^2 + 9x - 22 \\ & -2x^2 + 6x \\ \hline & 15x - 22 \\ & -15x + 45 \\ \hline & 23 \end{array}$$

HOMEWORK

1. Factor:
a. $6a + 12 =$
b. $mn + n =$
c. $5xy - 15x$=
d. $4ax - 8ax^2 + 12ax^3 =$

2. Factor using the factorization identities we learned above:
a. $9 - x^2 =$
b. $x^6 - 4 =$
c. $9 - 6x + x^2 =$
d. $a^3 - 2a^2x + ax^2 =$

3. Show that the left hand side (LHS) = right hand side (RHS):
a. $(m-n)(a+b) + m - n = a(m-n) + (b+1)(m-n)$
b. $x^2(x+1) - x - 1 = x(x+1)^2 - (x+1)^2$
c. $2x(x+b) + a(x+b) = (2x+a)x + (2x+a)b$
d. $(a+b)^2 + c(a+b) = (a+b)(a+c) + (a+b)b$

4. Complete the long division of polynomials:
(a) $x^2 - 3x - 4$ by $x - 4$
(b) $x^3 - 2x^2 + 2x - 4$ by $x - 2$
(c) $x^4 + 3x^3 - x^2 - x + 6$ by $x + 3$
(d) $2x^4 - 5x^3 + 2x^2 + 5x - 10$ by $x - 2$

5. Solve the following equations by first guessing one of the solutions, and then using polynomial division.
(a) $2x^2 + 3x - 14 = 0$
(b) $3x^2 - 10x + 3 = 0$
(c) $5x^2 + 8x - 4 = 0$

6. Solve the following inequalities:

(a) $\frac{x}{x+1} > 2$

(b) $(1-x)(2x+1) < 0$