

## MATH 6: HANDOUT 22

### SOLVING EQUATIONS AND INEQUALITIES

#### SOLVING INEQUALITIES INVOLVING NEGATIVE NUMBERS

Today we discussed inequalities and their solutions, discussing how one solves inequalities involving negative numbers. The rule here is

$$a < b \iff (-a) > (-b)$$

(note that the sign of inequality is reversed!). For example,  $3 < 5$ , but  $-3 > -5$ .

More generally, if we multiply or divide both sides of an inequality by a negative number, we need to change the sign of inequality, replacing  $<$  by  $>$  and vice versa. For example, to solve  $-3x > -6$ , we divide both sides by  $-3$  and change  $>$  to  $<$ , giving  $x < 2$ .

#### PRODUCTS

The following rules are frequently used when dealing with equations or inequalities where the left-hand side is a product of two factors:

- A product of two numbers is zero if and only if one of them is zero:

$$ab = 0 \iff (a = 0) \text{ OR } (b = 0)$$

- A product of two numbers is positive if and only if both numbers are positive or both numbers are negative:

$$ab > 0 \iff (a > 0 \text{ AND } b > 0) \text{ OR } (a < 0 \text{ AND } b < 0)$$

- A product of two numbers is negative if and only if one of numbers is positive and the other one is negative:

$$ab < 0 \iff (a > 0 \text{ AND } b < 0) \text{ OR } (a < 0 \text{ AND } b > 0)$$

#### HOMEWORK

1. (This problem is for students who have some difficulty with negative numbers and need extra practice).

Compute the following expressions:

$$\begin{array}{lll} -3 - (-2) & -3 - 3 & ((-4) \times 6) - 7 \\ (-6) \div (-2) + 3 & (-2) \div (-3) & (-4) \times (-7) \div (+9) \\ (-4) - (-3) + (-5) & (-6) + (-2) \div (-3) & \end{array}$$

2. Solve the following inequalities

$$\begin{array}{lll} \text{(a)} -x < 2 & \text{(b)} 2 - 3x > 5 & \text{(c)} 3x + 1 < 5x + 7 \\ \text{(d)} 1 + 5x < 3x & \text{(e)} x - 1 < x - 7 & \end{array}$$

3. Solve the following equations and inequalities:

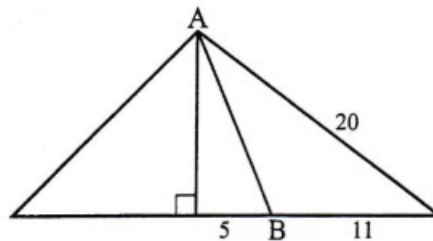
$$\begin{array}{lll} \text{(a)} (x - 1)(x - 2) = 0 & \text{(b)} x(x + 1) < 0 & \text{(c)} \frac{1}{x} > 2 \\ \text{(d)} x^2 - 4 = 0 & \text{(e)} \frac{x}{x + 1} > 1 & \end{array}$$

[**Hints:** In (c), multiple the inequality by  $x$ . You need to be careful, whether  $x$  is positive or negative. In (e), consider separately two cases:  $x + 1 > 0$  and  $x + 1 < 0$ .]

4. In the figure below, each symbol stands for a number. The sum of numbers in each column or row is written next to the column or row — except for the second column, where the sum is not known. Can you find this missing sum?

⊙	☆	▲	▲	96
▲	⊙	⊙	⊙	92
☆	☆	☆	▲	140
⊙	▲	⊙	☆	108
108	?	108	96	

5. Old MacDonald raises sheep and chickens on his farm. His livestock has a total of 55 heads and 142 legs among them (not counting the farmer!). How many chickens and how many sheep does he have?
6. Mr. Sim jogs at 9 km per hour over a certain distance and walks back the same distance at 6 km per hour. What is his average speed? [Hint: the answer  $(6 + 9)/2 = 7.5$  is wrong!].
7. Show that in a class of 40 students, there must be two students whose birthdays are less than 10 days apart. ‘
8. January 1, 2018 was a Monday. What is the next year in which January 1 will fall on a Monday? (Mathcounts)
9. Find AB (Mathcounts).



10. Three cubes are stacked as shown. If the cubes have edge 1, 2 and 3 what is AB? (Mathcounts)

