Math 5e, Homework 25

due April 23

Instructions: Some of the problems we solved in class, and some are new. Please try to solve all problems, do your best, and show your work. Write on separate sheets of paper, not between the lines of this handout!

Geometry: Congruency

Two figures are called congruent if they have the same shape and size. We use the symbol ≅ to denote congruent figures: to say that M_1 is congruent to M_2, one writes $M_1 \cong M_2$.

The precise definition of the "same shape and size" depends on the figure. Most importantly, for triangles, it means that corresponding sides are equal and corresponding angles are equal: $\Delta ABC \cong \Delta A'B'C'$ is the same as: $AB = A'B'$, $BC = B'C'$, $AC = A'C'$,

$$\angle A = \angle A', \quad \angle B = \angle B', \quad \angle C = \angle C'.$$

- Note that for triangles, the notation $\Delta ABC \cong \Delta A'B'C'$ tells that these two triangles are congruent and also shows which vertex of the first triangle corresponds to which vertex of the second one. For example, $\Delta ABC \cong \Delta PQR$ is not the same as $\Delta ABC \cong \Delta QPR$.

Congruent triangles

Rule 1 (Side-Side-Side rule). If $AB \cong A'B'$, $BC \cong B'C'$ and $AC \cong A'C'$ then $\Delta ABC \cong \Delta A'B'C'$.

This rule is commonly referred to as the ***SSS*** rule.

One can also try other ways to define a triangle by three pieces of information, such as two sides and an angle between them. We will discuss it next time. This rule – and congruent triangles in general – are very useful for proving various properties of geometric figures.

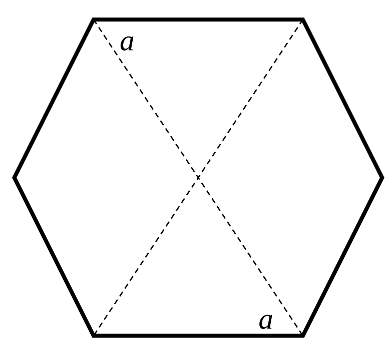
Parallelogram: A parallelogram is a quadrilateral in which opposite sides are parallel.

Sum of angles of an n-gon: is (n - 2) × 1800.

Homework

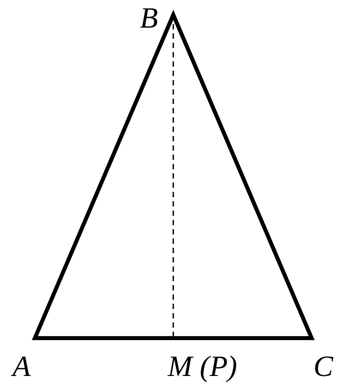
(some problems were solved in class; review notes and please solve again)

1. An *n-gon* is called *regular* if all sides are equal and all angles are also equal.
   (a) How large is each angle in a regular hexagon (6-*gon*)?
   (b) Show that in a regular hexagon, opposite sides are parallel. (This is why this shape is used for nuts and bolts).
   [Hint: show that each of the angles labeled by the letter *a* in the figure is equal to 60°, and then use the theorem about alternate interior angles.]



2. Let ABC be a triangle in which two sides are equal: $AB = BC$ (isosceles triangle). We proved in class that if M is the midpoint of the side AB, i.e. $AM = MB$, then
   - ✓ triangles AMC and BMC are congruent.
   - ✓ angles A and B are equal
   - ✓ angle $AMC = 90$ degrees

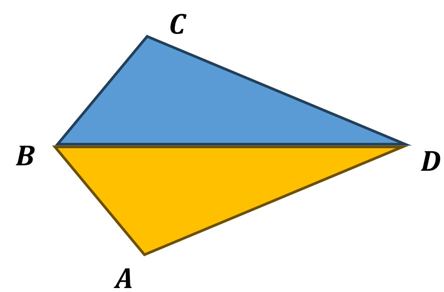
   So, in an isosceles, the median is also a height.



   a) Please review your notes and prove the above 3 points again!

   b) Can you prove the following: If in the triangle ΔABC $\angle A = \angle B$ and the point P of the side AB is such that CP is a height ($\angle APC = 90^o$) then this triangle is isosceles?

3. For the sides of the quadrilateral ABCD is given that $AB = BC$ and $AD = CD$. Show (prove) that:
   a) $\triangle ABD \cong \triangle CBD$
   b) BD splits the angles $\angle ABC$ and $\angle ADC$ into two equal parts (an angle bisector)



4. Simplify the expressions:
   (a) $x - (1 + 5x) =$
   (b) $2x - (3x^2 + x - 1) + (2 + 2x - x^2) =$
   (c) $3x \cdot (-2xy) =$
   (d) $(y - 5)(y - 1) - (y + 2)(y - 3) =$
   (e) $3(x - 1)^2 - 3x(x - 5) =$