Topics: Equations, Introduction to probability

- Use the power rules; similarly to $(ab)^n = a^n b^n$, similarly $\sqrt{ab} = \sqrt{a}\sqrt{b}$.
  $a^m a^n = a^{m+n}$ and $a^{m+n} = a^m a^n$, similarly $\sqrt{a}\sqrt{a} = a^{1/2} a^{1/2} = a^{\frac{1}{2}+\frac{1}{2}} = a$
  $(a^m)^n = a^{m \times n}$, similarly $\left(\sqrt{a}\right)^2 = \left(a^{1/2}\right)^2 = a$

- Theorem (Pythagorean theorem). In a right triangle with legs (sides) a, b, and hypotenuse c, one has:
$$a^2 + b^2 = c^2$$
$$c = \sqrt{a^2 + b^2}$$

- Formulas for fast multiplication
$$(a + b)^2 = a^2 + 2ab + b^2$$
$$(a - b)^2 = a^2 - 2ab + b^2$$
$$(a - b)(a + b) = a^2 - b^2$$

Probability:

In general, the probability of obtaining one outcome from a certain collection of A possible outcomes, is given by
$$P(A) = \frac{Number\ of\ outcomes\ giving\ A}{total\ number\ of\ possible\ outcomes}$$

Addition rule:
Probability of outcome A to happen or outcome B to happen is sum of the individual probabilities
$$P(A or B) = P(A) + P(B)$$

Complement rule:
The sum of probability of all outcomes is 1. Then the probability of getting an outcome different from A is
$$(not A) = 1 - P(A)$$

Do it on your own

1. Solve the equations.
   a) $2x - x(x - 3) = 5 - x^2$

   b) $(3x - 1)(2x + 7) = (x + 1)(6x - 5)$

P r o b l e m s

2. Solve the equations.

a) $|4x - 5| = 25$

b) $|-x - 1| = 1$

c) $|x + 2| = -1$

2. Probability in a six-sided die

a) List all possible outcomes when rolling a die.

b) Calculate P(∷) , P($\cdot$ *or* ∷), P($\cdot\cdot$ *or* $\cdot\cdot\cdot$)

c) What is the probability to roll an even number?

3. Introducing the main probability rules

4. The standard card deck has 4 suits (hearts, diamonds, spades, and clubs - ♥ ♦ ♠ ♣);
each suit has 13 different card values: 2 through 10, jack, queen, king, and ace. If you randomly draw
one card, what is the probability of drawing
   a) number 6
   b) a red car
   c) a King or a Quin
   d) a red card and any Quin

5. Group work. You are given either a 9- sided die, a 10-sided die, and a 12 sided die. What is the
probability to roll
   a) P(3)
   b) P(even)
   c) P(sum = 4)

6. If I have $m = 9$ holes to put $n = 10$ pigeons, at least
one hole must contain 2 pigeons. This is known as the
Pigeonhole principle. It could be applied to many
problems where if there are more objects than containers,
at least one container must have more than one object.
In the general case, if we have n number of objects and
m number of places where $n > m$, at least $m - n$ object
have to be in one place, or find the remainder m/n

THE PIGEONHOLE PRINCIPLE
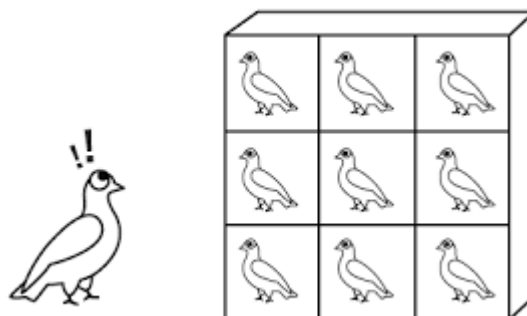
Applications:
a) In a group of 3 people, at least 2 must be the same gender.
b) In a class there are 13 students, at least 2 have the same birthday month.
c) If 11 people each shake hands with 12 others, what can we conclude?

7. Using the Pigeonhole principles, state what you can conclude about the following situations.
    a) For every 27-word sequence in the US Constitution, at least 2 words will ….
    b) If you pick 5 cards from a standard deck, at least 2 will be …..