MATH 5e: Class Work 18

Equations, rational equations, exponential equations

- Use the power rules; similarly to $(ab)^n = a^n b^n$, similarly $\sqrt{ab} = \sqrt{a}\sqrt{b}$.
  $a^m a^n = a^{m+n}$ and $a^{m+n} = a^m a^n$, similarly $\sqrt{a}\sqrt{a} = a^{1/2} a^{1/2} = a^{\frac{1}{2}+\frac{1}{2}} = a$
  $(a^m)^n = a^{m \times n}$, similarly $\left(\sqrt{a}\right)^2 = \left(a^{1/2}\right)^2 = a$

- Theorem (Pythagorean theorem). In a right triangle with legs (sides) a, b and hypotenuse c, one has:
$$a^2 + b^2 = c^2$$
$$c = \sqrt{a^2 + b^2}$$

- Formulas for fast multiplication
$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a-b)^2 = a^2 - 2ab + b^2$$
$$(a-b)(a+b) = a^2 - b^2$$

Do it on your own

1. In the triangle ΔABC the sides AC and BC are perpendicular to each other. If angle C has a measure in degrees of $\angle C = (3x)°$, and the sides $BC = x + 20$ and $AC = 2x - 20$, what type of triangle is this? Be specific.

Problems

1. Open the parentheses and solve. Is there are faster way to do this?

   a) $(4 - 3x)^2 =$

   b) $(5 + y)^2 =$

   c) $(3y - z)(3y + z) =$

   d) $(6 + x)^2 - (6 - x)(x + 6) =$

2. Simplify the following expressions using the formulas for fast multiplication.

   a) $\dfrac{x^2 - x}{x - 1} =$

b) $\dfrac{5+y}{25+50y+y^2} =$

c) $\dfrac{4-x^2}{x+4} =$

3. Open the parentheses and solve the following equations:

   a) $2x(x-1) = 2(x^2-5)$

   b) $(3x-1)(2x+7) = (x+1)(6x-5)$

   c) $(x+1)^2 - x(x+2) = 4$

4. Solve the equations:

   a) $\dfrac{7(x-1)}{(x+1)(x-1)} = 3$

   b) $\dfrac{3}{8}y - \dfrac{7}{8} - \dfrac{1}{6}(1-y) = 1\dfrac{5}{12}$

5. The perimeter of an isosceles triangle is 42. One of the triangle's sides is 6 cm shorter than the base.
   a) Find the length of all sides in this triangle.
   b) Find the height of the triangle and its area

6. In a children's fairytale, a worker earns 20 gold coins per. Even if he is not working, he must spend 6 gold coins daily for food and shelter. After 30 days, he ended up with no coins. How many days did he actually work?

7. Find the power
   a) $2^n = 8$

   b) $2^n = \dfrac{1}{8}$

   c) $2^{-n} = 8$

   d) $2^{-n} = \dfrac{1}{8}$

   e) $(-2)^n = -8$

   f) $(-2)^n = -\dfrac{1}{8}$

8. Find the power n

   a) $5^{1-n} = 5$

   b) $5^{1-n} = 25$

   c) $16^n = 32^{n-8}$

9. Write down a series of 7 numbers; the first number is 8^{-3} and every following number is a product of the previous number multiplied by 8. Then, find the product of
   a) The first number and the seventh
   b) The second and the sixth
   c) The third and the fifth
   d) All seven numbers

10. Write in scientific notation
   a) $10 \cdot 10^6 =$

   b) $0.5 \: x10^{-8} =$

   c) $17.5 \cdot 10^6 =$

   d) $15\,000\,000 =$

   e) $254\,000\,000 =$

   f) $0.00000009 =$

   g) $0.000000723 \: =$